Date: 10/10/2021

Q1. Find the number of terms in the expansion of $\left(1+5\sqrt{2}\,x\right)^9+\left(1-5\sqrt{2}\,x\right)^9$.

Q2. Find the total number of terms in the expansion of $\left(\sqrt{x}+y\right)^{10}+\left(\sqrt{x}-y\right)^{10}$.

Q3. Find the total number of terms in the expansion of $(x+\sqrt{y})^{10}+(x-\sqrt{y})^{10}$.

Q4. Find the number of terms in the expansion of $(x+y)^{2n}+(x-y)^{2n}$ $n \in N$.

Q5. Find the number of terms in the expansion of $(x+y)^2+(x-y)^2$.

Q6. Find the total number of terms in the expansion of $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^{30}$.

Q7. Find the total number of terms in the expansion of $\left(\sqrt{x}+\sqrt{y}\right)^{10}+\left(\sqrt{x}-\sqrt{y}\right)^{10}$.

Q8. Find the number of terms in the expansion of $(x^2+4x+4)^{30}$.

Q9. Find the number of terms in the expansion of $(1+x)^n \cdot (1-x)^n$, $N \in I$.

Q10. Find the number of terms in the expansion of $(1+2x+x^2)^{20}$.

Q11. Find the number of terms in the expansion of $(x^2+1)^{40}$.

Q12. Find the number of terms in the expansion of $\left(x-\frac{1}{x}\right)^{30}$.

Q13. Find the total number of terms in the expansion of $\left(\sqrt{x}+y\right)^{11}+\left(\sqrt{x}-y\right)^{11}$.

Q14. Find the total number of terms in the expansion of $(a+b)^{2k+1}+(a-b)^{2k+1}$ $(K \in I)$.

Q15. Find the total number of terms in $(a-b)^3+(a+b)^3$.

Q16. Find the total number of terms in the expansion of $\left(x+\sqrt{y}\right)^{11}+\left(x-\sqrt{y}\right)^{11}$.

Q17. Find the total number of terms in the expansion of $\left(\sqrt{x}+y\right)^9+\left(\sqrt{x}-y\right)^9$.

Q18. Find the number of terms in the following expansion.
(a) $(x+2y)^{10}$ (b) $(2x+y)^{10}$.

Q19. Find the number of terms in the expansion of $(x^2+4x+4)^2+(x^2-4x+4)^2$.

Q20. Find the number of terms in the expansion of $(1+2x+x^2)^5\,(1-2x+x^2)^5$.

Q21. Find the total number of terms in the expansion of $(1+2x+x^2)^n+(1-2x+x^2)^n$, $n \in I$.

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S1. Since if n is odd, then expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. Hence total number of terms in the given expression is $\left(\frac{9+1}{2}\right)$ = 5 terms.

S2. Since if n is even then the expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n}{2}+1\right)$ terms. Hence total number of terms in the given expansion is $\left(\frac{10}{2}+1\right)$ = 6 terms.

S3. Since if n is even, then the expansion of $(a + b)^n + (a - b)^n$ Contains $\left(\frac{n}{2}+1\right)$ terms. Hence total no. of terms in the required expression is $\left(\frac{10}{2}+1\right) = 6$ terms .

S4. We know that if n is even, then expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n}{2}+1\right)$ terms. Hence required number of terms in the following expansion is $\left(\frac{2n}{2}+1\right)$ = $(n + 1)$ terms.

S5. We know that if n is even then expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n}{2}+1\right)$ terms.

Hence required number of terms in the given expansion is $\left(\frac{2}{2}+1\right)$ = 2 terms.

Alternate Method:

$(x + y)^2 + (x + y)^2 = 2(x^2 + y^2)$.

Clearly it contains two terms.

S6. We know that the number of terms in the expansion of $(a + b)^n$ is $(n + 1)$.

Hence total number of terms in this expansion is $(30 + 1) = 31$.

S7. We know that if n is even than $(a + b)^n + (a - b)^n$ contains $\left(\frac{n}{2}+1\right)$ terms hence required number of terms in the following expansion is $\left(\frac{10}{2}+1\right) = 6$ terms.

S8. $(x^2 + 4x + 4)^{30}$ can be written as. $((x + 2)^2)^{30} = (x + 2)^{60}$.

Hence total number of terms in this given expansion is $60 + 1 = 61$.

S9.
$$(1 + x)^n . (1 - x)^n = (1 - x^2)^n.$$

Hence total number of terms in the given expansion is $(n + 1)$.

S10. $\therefore$ $$(1 + 2x + x^2)^{20} = ((1 + x)^2)^{20} = (1 + x)^{40}.$$

Hence total number of terms in the given expansion is 40 + 1 = 41.

S11. We know that the number of terms in the expansion of $(a + b)^n$ is $(n + 1)$. Hence number of terms in the expansion of $(x^2 + 1)^{40}$ is 40 + 1 = 41.

S12. We know that the number of terms in the expansion of $(a + b)^n$ is $(n + 1)$. Hence number of terms in $\left(x - \frac{1}{x}\right)^{30}$ is 30 + 1 = 31.

S13. Since if n is odd, then expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. Hence total number of terms in the given expression is $\left(\frac{11+1}{2}\right) = 6$ terms.

S14. $K \in I$, $(2k + 1)$ is an odd integer.

Hence total number of terms in the following expression is $\left(\frac{2k+1+1}{2}\right) = (k+1)$ terms.

S15. Since if n is odd then expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. Hence total number of terms in the given expression is $\left(\frac{3+1}{2}\right) = 2$ terms.

S16. If n is odd then expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. Hence, required number of terms. in the following expansion is $\left(\frac{11+1}{2}\right) = 6$ terms.

S17. Since if n is odd, then expansion of $(a + b)^n + (a - b)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. Hence total number of terms in the given expansion is $\left(\frac{9+1}{2}\right) = 5$ terms.

S18. We know that the number of terms in the expansion of $(a + b)^n$ is $(n + 1)$ hence.

(a) The number of terms in $(x + 2y)^{10}$ is 10 + 1 = 11.

(b) The number of terms in $(2x + y)^{10}$ is 10 + 1 = 11.

S19. $$(x^2 + 4x + 4) = (x + 2)^2.$$
$$(x^2 - 4x + 4) = (x - 2)^2.$$

Hence required expression can be written as

$$((x + 2)^2)^2 + ((x - 2)^2)^2 = (x + 2)^4 + (x - 2)^4.$$
$$= (x + 2)^4 + (x - 2)^4.$$

Hence total number of terms in this given expression is $\left(\frac{4}{2} + 1\right) = 3$ terms.

S20. $(1 + 2x + x^2)^5 = ((1 + x)^2)^5 = (1 + x)^{10}.$

$(1 - 2x + x^2)^5 = ((1 - x)^2)^5 = (1 - x)^{10}.$

$\therefore \quad (1 + 2x + x^2)^5 \cdot (1 - 2x + x^2)^5 = (1 + x)^{10}(1 - x)^{10} = (1 - x^2)^{10}$

Hence total no. of terms in this expansion is 10 + 1 = 11.

S21. $(1 + 2x + x^2)^n = (1 + x)^{2n}.$

$(1 - 2x + x^2)^n = ((1 - x)^2)^n = (1 - x)^{2n}$

Hence required expression can be written as $(1 + x)^{2n} + (1 - x)^{2n}$.

Hence total number of terms in this given expression is $\left(\frac{2n}{2} + 1\right) = (n + 1)$.

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Q1. Using binomial theorem expand the following: $(x + 3y)^3$.

Q2. Using binomial theorem expand the following: $(2x + 3y)^4$.

Q3. Using binomial theorem expand the following; $(3x^2 - 2y)^4$.

Q4. Using binomial theorem expand the following: $(1 - x + x^2)^4$.

Q5. Prove that,

$$\sum_{r=0}^{n} a^r \, {}^nC_r = (a+1)^n.$$

Q6. Expand the following expressions: $(1 - 2x)^5$.

Q7. Expand $\left(x^2 + \frac{3}{x}\right)^4, x \neq 0$.

Q8. Expand the following expression $(1 + x + x^2)^3$.

Q9. Using binomial theorem expand $\{(x + y)^5 - (x - y)^5\}$ and hence find the value of $\{(\sqrt{3}+1)^5 - (\sqrt{3}-1)^5\}$.

Q10. Evaluate: $(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6$

Q11. Using binomial theorem write down expansions of the following:

$$\left(x + 1 - \frac{1}{x}\right)^3$$

Q12. Evaluate the following expression $(\sqrt{2}+1)^6 - (\sqrt{2}-1)^6$.

Q13. Using binomial theorem expand the following:

$$\left(2x - \frac{1}{x}\right)^5$$

Q14. Prove that by expansion:

$$\sum_{r=0}^{n} 3^r \, {}^nC_r = 4^n.$$

Q15. Find the expansion of $(3x^2 - 2ax + 3a^2)^3$.

Q16. Expand the following expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$.

Q17. Expand the following expression $(1 - x)^6$.

Q18. Expand $(x^2 + 2y)^5$ by binomial theorem.

Q19. Expand using binomial theorem

$$\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4, x \neq 0.$$

Q20. Expand the following expressions:

$$\left(x-\frac{1}{y}\right)^{11}, y \neq 0$$

Q21. Evaluate the following: $(\sqrt{x+1}+\sqrt{x-1})^6+(\sqrt{x+1}-\sqrt{x-1})^6$

Q22. Using binomial theorem write down the expansions of the following:

$$\left(\frac{2}{y}-\frac{y}{2}\right)^{8}, y \neq 0$$

Q23. Using binomial theorem write down the expansions of the following:

$$\left(\frac{x}{3}-\frac{4}{3x^2}\right)^{5}, x \neq 0$$

Q24. Using binomial theorem write down the expansions of the following:

$$\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^{6}$$

Q25. Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4$.

Q26. Find the value of : $(a^2+\sqrt{a^2-1})^4+(a^2-\sqrt{a^2-1})^4$.

Q27. Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$.

Q28. Expand the following expressions: $(2x - 3)^6$.

Q29. Expand the following expressions: $\left(x+\frac{1}{x}\right)^6$.

Q30. Expand the following expressions: $\left(\frac{x}{3}+\frac{1}{x}\right)^5$.

Q31. Using binomial theorem write down the expansions of the following: $(1 + 2x - 3x^2)^5$

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S1.
$$(x + 3y)^3 = {}^3C_0x^3 + {}^3C_1x^2(3y)^1 + {}^3C_2x^1(3y)^2 + {}^3C_3(3y)^3$$
$$= x^3 + 9x^2y + 27xy^2 + 27y^3$$

S2.
$$(2x + 3y)^4 = {}^4C_0(2x)^4 + {}^4C_1(2x)^3(3y) + {}^4C_2(2x)^2(3y)^2 + {}^4C_3(2x)(3y)^3 + {}^4C_4(3y)^4$$
$$= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$$

S3.
$$(3x^2 - 2y)^4 = {}^4C_0(3x^2)^4 + {}^4C_1(3x^2)^3(-2y) + {}^4C_2(3x^2)^2(-2y)^2 + {}^4C_3(3x^2)(-2y)^3 + {}^4C_4(-2y)^4$$
$$= 81x^8 - 216x^6y + 216x^4y^2 - 16x^2y^3 + 16y^4$$

S4.
$$(1 - x + x^2)^4 = [(1 - x) + x^2]^4$$
$$= {}^4C_0(1 - x)^4 + {}^4C_1(1 - x)^3(x^2) + {}^4C_2(1 - x)^2(x^2)^2 + {}^4C_3(1 - x)(x^2)^3 + {}^4C_4(x^2)^4$$
$$= (1 - 4x + 6x^2 + x^4) + 4(1 - 3x + 3x^2 - x^3)x^2 + 6(1 - 2x + x^2)x^4 + 4(1 - x)x^6 + x^8$$
$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^5 + 10x^6 - 4x^7 + x^8$$

S5.
$$\sum_{r=0}^{n} a^r\, {}^nC_r = a^0\, {}^nC_0 + a^1\, {}^nC_1 + a^2\, {}^nC_2 + \ldots + a^n . {}^nC_n$$
$$= a^0\, {}^nC_0 + a^1\, {}^nC_1 + a^2\, {}^nC_2 + \ldots + a^n\, {}^nC_n$$
$$= 1 + {}^nC_1.a + {}^nC_2\, a^2 + \ldots\ldots + {}^nC_n\, a^n = (a + 1)^n.$$

S6. By using Binomial Theorem, we have
$$(1 - 2x)^5 = [1 + (-2x)]^5$$
$$= {}^5C_0 + {}^5C_1(-2x) + {}^5C_2(-2x)^2 + {}^5C_3(-2x)^3 + {}^5C_4(-2x)^4 + {}^5C_5(-2x)^5$$
$$= 1 + 5(-2x) + 10(-2x)^2 + 10(-2x)^3 + 5(-2x)^4 + (-2x)^5$$
$$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5.$$

S7. By using binomial theorem, we have
$$\left(x^2 + \frac{3}{x}\right)^4 = {}^4C_0(x^2)^4 + {}^4C_1(x^2)^3 \cdot \left(\frac{3}{x}\right) + {}^4C_2(x^2)^2\left(\frac{3}{x}\right)^2 + {}^4C_3(x^2)\left(\frac{3}{x}\right)^3 + {}^4C_4\left(\frac{3}{x}\right)^4$$
$$= x^8 + 4 \cdot x^6 \cdot \frac{3}{x} + 6 \cdot x^4 \cdot \frac{9}{x^2} + 4 \cdot x^2 \cdot \frac{27}{x^3} + \frac{81}{x^4}$$
$$= x^8 + 12x^5 + 54x^2 + \frac{108}{x} + \frac{81}{x^4}.$$

S8. Let $(x + x^2) = y.$

$\therefore$ $(1 + x + x^2)^3 = (1 + y)^3.$

$= {}^3C_0 + {}^3C_1\, y + {}^3C_2\, y^2 + {}^3C_3\, y^3$

$= 1 + 3y + 3y^2 + y^3$

$= 1 + 3\,(x + x^2) + 3\,(x + x^2)^2 + (x + x^2)^3.$

$= 1 + 3\,(x + x^2) + 3\,(x^2 + 2x^3 + x^4) + (x^3 + 3x^4 + 3x^5 + x^6)$

$= x^2 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1.$

S9. We have, $(x + y)^5 - (x - y)^5 = 2[{}^5C_1x^{5-1}y + {}^5C_3x^{5-3}y^3 + {}^5C_5x^{5-5}y^5]$

$$= 2\left[\frac{5!}{1!4!}x^4y + \frac{5!}{3!2!}x^2y^3 + \frac{5!}{5!0!}x^0y^5\right]$$

$\Rightarrow$ $(x + y)^5 - (x - y)^5 = 2[5x^4y + 10x^2y^3 + y^5]$ ... (i)

Putting $x = \sqrt{3}$ and $y = 1$ in Eq. (i), we get

$$\left(\sqrt{3}+1\right)^5 - \left(\sqrt{3}-1\right)^5 = 2\left[5(\sqrt{3})^4(1) + 10(\sqrt{3})^2(1)^3 + (1)^5\right]$$

$$= 2\,[45 + 30 + 1] = 2 \times 76 = 152$$

S10. We have, $(a + b)^n - (a - b)^n = 2[{}^nC_1\, a^{n-1}\, b + {}^nC_3\, a^{n-3}\, b^3 + ...]$

Putting $a = \sqrt{3}, b = \sqrt{2}$ and $n = 6$ in the above, we get

$$\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6 = 2\left[{}^6C_1\left(\sqrt{3}\right)^5\left(\sqrt{2}\right)^1 + {}^6C_3\left(\sqrt{3}\right)^{6-3}\left(\sqrt{2}\right)^3 + {}^6C_5\left(\sqrt{3}\right)^{6-5}\left(\sqrt{2}\right)^5\right]$$

$$= 2[6(9\sqrt{3})(\sqrt{2}) + 20(3\sqrt{3})(2\sqrt{2}) + 6(\sqrt{3})(4\sqrt{2})]$$

$$= 2[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6}] = 396\sqrt{6}$$

S11. $(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + ... + {}^nC_nb^n.$

$$\left(x+1-\frac{1}{x}\right)^3 = {}^3C_0(x + 1)^3 + {}^3C_1(x + 1)^2 \times \left(-\frac{1}{x}\right) + 3C_2(x+1)\left(-\frac{1}{x}\right)^2 + {}^3C_3\left(-\frac{1}{x}\right)^3$$

$$\left(x+1-\frac{1}{x}\right)^3 = x^3 + 1 + 3x^2 + 3x - \frac{3}{x} - 3x - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3} \qquad \left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$\left(x+1-\frac{1}{x}\right)^3 = x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3}.$$

S12. We know that

$$(a+b)^n - (a-b)^n = 2[{}^nC_1 a^{n-1} b + {}^nC_3 a^{n-3} b^3 + \ldots\ldots]$$

$$\left(\sqrt{2}+1\right)^6 - \left(\sqrt{2}-1\right)^6 = 2\left[{}^6C_1\left(\sqrt{2}\right)^5(1) + {}^6C_3\left(\sqrt{2}\right)^3(1)^3 + {}^6C_5\left(\sqrt{2}\right)(1)^5\right]$$

$$= 2\left[6\times 4\sqrt{2} + 20(2\sqrt{2}) + 6\sqrt{2}\right]$$

$$= 2\left[24\sqrt{2} + 40\sqrt{2} + 6\sqrt{2}\right] = 2(70\sqrt{2}) = 140\sqrt{2}\,.$$

S13.

$$\left(2x - \frac{1}{x}\right)^5 = {}^5C_0(2x)^5 + {}^5C_1(2x)^4\left(-\frac{1}{x}\right) + {}^5C_2(2x)^3\left(-\frac{1}{x}\right)^2$$

$$+ {}^5C_3(2x)^2\left(-\frac{1}{x}\right)^3 + {}^5C_4(2x)\left(-\frac{1}{x}\right)^4 + {}^5C_5\left(-\frac{1}{x}\right)^5$$

$$= 32x^5 - 80x^3 + 80x - 40\cdot\frac{1}{x} + 10\cdot\frac{1}{x^3} - \frac{1}{x^5}\,.$$

S14.

$$\sum_{r=0}^{n} 3^r\, {}^nC_r = 3^0\, {}^nC_0 + 3^1\, {}^nC_1 + 3^2\, {}^nC_2 + \ldots + 3^n\,.\,{}^nC_n$$

$$= 1 + {}^nC_1(3)^1 + {}^nC_2\,(3)^2 + {}^nC_3\,(3)^3 + \ldots\ldots\ldots.$$

$$= 1 + {}^nC_1(3)^1 + {}^nC_2\,(3)^2 + {}^nC_3\,(3)^3 + \ldots\ldots.. + {}^nC_n(3)^n$$

$$= (1+3)^n = 4^n.$$

S15. $(3x^2 - 2ax + 3a^2)^3$

$$= \{3x^2 - a(2x - 3a)\}^3$$

$$= {}^3C_0\,(3x^2)^3 - 3C_1\,(3x^2)^2\,\{a(2x-3a)\} + {}^3C_2\,(3x^2)\,a^2\,(2x-3a)^2$$
$$- 3C_3\,a^3(2x-3a)^3$$

$$= (3x^2)^3 - 3\,(3x^2)^2\,a(2x-3a) + 3(3x^2).a^2\,(2x-3a)^2 - a^3\,(2x-3a)^3$$

$$= 27x^6 - 9x^4\cdot 3a(2x-3a) + 9x^2\,a^2(4x^2 - 12ax + 9a^2y) - a^3$$
$$(8x^3 - 36x^2a + 54xa^2 - 27a^3$$

$$= 27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^2x + 27a^6.$$

S16. Since, $(a+b)^n = {}^nC_0\,a^n + {}^nC_1\,a^{n-1}\,b + {}^nC_2\,a^{n-2}\,b^2 + \ldots\ldots\ldots\ldots\ldots\ldots$

$\therefore$

$$\left(\frac{2}{x} - \frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^5 + {}^5C_1\left(\frac{2}{x}\right)^4.\left(\frac{-x}{2}\right)^1 + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{-x}{2}\right)^2$$

$$+ {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{-x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)^1\left(\frac{-x}{2}\right)^4 + {}^5C_5\left(\frac{-x}{2}\right)^5.$$

$$= 1\left(\frac{2}{x}\right)^5 + 5\left(\frac{2}{x}\right)^4\left(\frac{-x}{2}\right) + 10\left(\frac{2}{x}\right)^3\left(\frac{-x}{2}\right)^2$$

$$+ 10\left(\frac{2}{x}\right)^2\left(\frac{-x}{2}\right)^3 + 5\left(\frac{2}{x}\right)\left(\frac{-x}{2}\right)^4 + \left(\frac{-x}{2}\right)^5$$

$$= 32x^{-5} - 40x^{-3} + 20x^{-1} - 5x + \frac{5}{8}x^3 - \frac{1}{32}x^5.$$

S17. We know that, $(a + b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}\, b + {}^nC_2\, a^{n-2} b^2 + \ldots.. + {}^nC_n\, b^n.$

$\therefore$ $(1 - x)^6 = {}^6C_0\, 1^6 + {}^6C_1\, 1^5\,(-x) + {}^6C_2\, 1^4\,(-x)^2 + {}^6C_3\, 1^3\,(-x)^3 + \ldots..$

$$= \frac{6!}{0!\cdot 6!}(1) + \frac{6!}{1!(6-1)!}(-x) + \ldots\ldots$$

$$= \frac{6!}{0!\cdot 6!} + \frac{6!}{1!(6-1)!}(-x) + \ldots\ldots\ldots\ldots$$

$$= 1 - 6x + 15\,x^2 - 20\,x^3 + 15x^4 - 6x^5 + x^6$$

S18. $(x^2 + 2y)^5 = {}^5C_0(x^2)^5 + {}^5C_1(x^2)^4\,(2y) + {}^5C_2(x^2)^3(2y)^2 + {}^5C_3\cdot(x^2)^2\,(2y)^3 + {}^5C_4\cdot x^2\,(2y)^4 + {}^5C_5\,(2y)^5$

$$= x^{10} + 5x^8(2y) + 10x^6(4y^2) + 10x^4\,8y^3 + 5x^2(16y^4) + 32y^5$$

$$= x^{10} + 10x^8y + 40\,x^6y^2 + 80x^4y^3 + 80x^2y^4 + 32y^5.$$

S19. We have,

$$\left(1+\frac{x}{2}-\frac{2}{x}\right)^4 = \left\{1+\left(\frac{x}{2}-\frac{2}{x}\right)\right\}^4$$

$$= {}^4C_0 1^4 + {}^4C_1\left(\frac{x}{2}-\frac{2}{x}\right) + {}^4C_2\left(\frac{x}{2}-\frac{2}{x}\right)^2 + {}^4C_3\left(\frac{x}{2}-\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{x}{2}-\frac{2}{x}\right)^4$$

$$= 1 + 4\left(\frac{x}{2}-\frac{2}{x}\right) + \frac{4(4-1)}{2!}\left(\frac{x}{2}-\frac{2}{x}\right)^2 + \frac{4(4-1)(4-2)}{3!}\left(\frac{x}{2}-\frac{2}{x}\right)^3$$

$$+ \frac{4(4-1)(4-2)(4-3)}{4!}\left(\frac{x}{2}-\frac{2}{x}\right)^4$$

$$= 1 + 4\left(\frac{x}{2}-\frac{2}{x}\right) + 6\left(\frac{x^2}{4}+\frac{4}{x^2}-2\right)$$

$$+ 4\left\{{}^3C_0\left(\frac{x}{2}\right)^3 + {}^3C_1\left(\frac{x}{2}\right)^2\left(-\frac{2}{x}\right)^1 + {}^3C_2\left(\frac{x}{2}\right)^1\left(-\frac{2}{x}\right)^2 + {}^3C_3\left(-\frac{2}{x}\right)^3\right\}$$

$$+4\left\{\left(\frac{x}{2}\right)^4+4\left(\frac{x}{2}\right)^3\left(-\frac{2}{x}\right)+\frac{4(4-1)}{2!}\left(\frac{x}{2}\right)^2\left(-\frac{2}{x}\right)^2\right.$$

$$\left.+\frac{4(4-1)(4-2)}{3!}\left(\frac{x}{2}\right)\left(-\frac{2}{x}\right)^3+\frac{4(4-1)(4-2)(4-3)}{4!}\left(-\frac{2}{x}\right)^4\right\}$$

$$=1+4\left(\frac{x}{2}-\frac{2}{x}\right)+6\left(\frac{x^2}{4}+\frac{4}{x^2}-2\right)$$

$$+4\left\{\frac{x^3}{8}-\frac{3x}{2}+\frac{6}{x}-\frac{8}{x^3}\right\}+\left\{\frac{x^4}{16}-x^2+6-\frac{16}{x^2}+\frac{16}{x^4}\right\}$$

$$=1+x(2-6)+x^2\left(\frac{3}{2}-1\right)+x^3\left(\frac{1}{2}\right)+x^4\left(\frac{1}{16}\right)+\frac{1}{x}(-8+24)$$

$$+\frac{1}{x^2}(24-16)+\frac{1}{x^3}(-32)+\frac{1}{x^4}(16)-12+6$$

$$=-5-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}+\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}.$$

S20. We have,

$$\left(x-\frac{1}{y}\right)^{11}={}^{11}C_0x^{11}\left(-\frac{1}{y}\right)^0+{}^{11}C_1x^{10}\left(-\frac{1}{y}\right)^1+{}^{11}C_2x^9\left(-\frac{1}{y}\right)^2+{}^{11}C_3x^8\left(-\frac{1}{y}\right)^3$$

$$+{}^{11}C_4x^7\left(-\frac{1}{y}\right)^4+{}^{11}C_5x^6\left(-\frac{1}{y}\right)^5+{}^{11}C_6x^5\left(-\frac{1}{y}\right)^6+{}^{11}C_7x^4\left(-\frac{1}{y}\right)^7$$

$$+{}^{11}C_8x^3\left(-\frac{1}{y}\right)^8+{}^{11}C_9x^2\left(-\frac{1}{y}\right)^9+{}^{11}C_{10}x^1\left(-\frac{1}{y}\right)^{10}+{}^{11}C_{11}x^0\left(-\frac{1}{y}\right)^{11}$$

$$=x^{11}-11\frac{x^{10}}{y}+55\frac{x^9}{y^2}-165\frac{x^8}{y^3}+330\frac{x^7}{y^4}-462\frac{x^6}{y^5}$$

$$+462\frac{x^5}{y^6}-\frac{330x^4}{y^7}+165\frac{x^3}{y^8}-55\frac{x^2}{y^9}+11\frac{x}{y^{10}}-\frac{1}{y^{11}}$$

$$=x^{11}-11x^{10}y^{-1}+55x^9y^{-2}-165x^8y^{-3}+330x^7y^{-4}-462x^6y^{-5}$$

$$+462x^5y^{-6}-330x^4y^{-7}+165x^3y^{-8}-55x^2y^{-9}+11xy^{-10}-y^{-11}.$$

S21.

$$(a+b)^n+(a-b)^n=2[{}^nC_0a^nb^0+{}^nC_2a^{n-2}b^2+{}^nC_4a^{n-4}b^4+\ldots]$$

$$(\sqrt{x+1}+\sqrt{x-1})^6+(\sqrt{x+1}-\sqrt{x-1})^6=2[{}^6C_0(\sqrt{x+1})^6+{}^6C_2(\sqrt{x+1})^4(\sqrt{x-1})^2$$

$$+{}^6C_4(\sqrt{x+1})^2(\sqrt{x-1})^4+{}^6C_6(\sqrt{x-1})^6]$$

$$=2[(x+1)^3+15(x+1)^2(x-1)+15(x+1)(x-1)^2$$

$$+(x-1)^3]$$

$$= 2[x^3 + 1 + 3x^2 + 3x + 15(x+1)(x+1)(x-1)$$
$$+ 15\,(x+1)(x-1)(x-1) + x^3 - 1 - 3x^2 + 3x]$$
$$= 2[x^3 + 3x + 15(x+1)(x^2-1) + 15\,(x-1)(x^2-1)$$
$$+ x^3 + 3x]$$

$$= 2[2x^3 + 6x + 15(x^3 - x + x^2 - 1) + 15(x^3 - x - x^2 + 1)]$$
$$= 2[2x^3 + 6x + 15x^3 - 15x + 15x^2 - 15 + 15x^3 - 15x - 15x^2 + 15\ ]$$
$$= 2[32x^3 - 24\,x] = 64x^3 - 48\,x.$$

S22. $$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}\,b + {}^nC_2a^{n-2}\,b^2 + \dots + {}^nC_nb^n$$

$$\left(\frac{2}{y}-\frac{y}{2}\right)^8 = {}^8C_0\left(\frac{2}{y}\right)^8 + {}^8C_1\left(\frac{2}{y}\right)^7\left(-\frac{y}{2}\right) + {}^8C_2\left(\frac{2}{y}\right)^6\left(-\frac{y}{2}\right)^2$$
$$+ {}^8C_3\left(\frac{2}{y}\right)^5\left(-\frac{y}{2}\right)^3 + {}^8C_4\left(\frac{2}{y}\right)^4\left(-\frac{y}{2}\right)^4 + {}^8C_5\left(\frac{2}{y}\right)^3\left(-\frac{y}{2}\right)^5$$
$$+ {}^8C_6\left(\frac{2}{y}\right)^2\left(-\frac{y}{2}\right)^6 + {}^8C_7\left(\frac{2}{y}\right)^1\left(-\frac{y}{2}\right)^7 + {}^8C_8\left(-\frac{y}{2}\right)^8$$

$$\left(\frac{2}{y}-\frac{y}{2}\right)^8 = \frac{256}{y^8} - 8\times\frac{128}{y^7}\cdot\frac{y}{2} + 28\times\frac{64}{y^6}\times\frac{y^2}{4} - 56\times\frac{32}{y^5}\times\frac{y^3}{8}$$
$$+ 70\times\frac{16}{y^4}\times\frac{y^4}{16} - 56\times\frac{8}{y^3}\times\frac{y^5}{32} + 28\times\frac{4}{y^2}\times\frac{y^6}{64}$$
$$- 8\times\frac{2}{y}\times\frac{y^7}{128} + \frac{y^8}{256}$$

$$\left(\frac{2}{y}-\frac{y}{2}\right)^8 = \frac{256}{y^8} - \frac{512}{y^6} + \frac{448}{y^4} - \frac{224}{y^4} - \frac{224}{y^2}$$
$$+ 70 - 14y^2 + \frac{7}{4}y^4 - \frac{y^6}{8} + \frac{y^8}{256}.$$

S23. $$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}\,b + {}^nC_2a^{n-2}\,b^2 + \dots + {}^nC_nb^n$$

$$\left(\frac{x}{3}-\frac{4}{3x^2}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(-\frac{4}{3x^2}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(-\frac{4}{3x^2}\right)^2$$
$$+ {}^5C_3\left(\frac{x}{3}\right)^2\left(-\frac{4}{3x^2}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(-\frac{4}{3x^2}\right)^4 + {}^5C_5\left(-\frac{4}{3x^2}\right)^5$$

$$= \frac{x^5}{243} + \frac{5.x^4}{81}\times\frac{(-4)}{3x^2} + 10\times\frac{x^3}{27}\times\frac{16}{9x^4} + \frac{10x^2}{9}\times\frac{-64}{27x^6}$$
$$+ 5.\frac{x}{3}\times\frac{256}{81x^8} + \frac{(-1024)}{243x^{10}}$$

$$\left[{}^nC_r = \frac{n!}{r!(n-r)}\right]$$

$$\left(\frac{x}{3}-\frac{4}{3x^2}\right)^5 = \frac{x^5}{243}-\frac{20}{243}x^2-\frac{5}{18x}-\frac{40}{27x^4}+\frac{320}{81x^7}-\frac{2024}{243x^{10}}.$$

S24. $(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + ... + {}^nC_nb^n$

$$\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6 = {}^6C_0\left(\sqrt{\frac{x}{a}}\right)^6 + {}^6C_1\left(\sqrt{\frac{x}{a}}\right)^5\left(-\sqrt{\frac{a}{x}}\right)+{}^6C_2\left(\sqrt{\frac{x}{a}}\right)^4\left(-\sqrt{\frac{a}{x}}\right)^2$$

$$+ {}^6C_3\left(\sqrt{\frac{x}{a}}\right)^3\left(-\sqrt{\frac{a}{x}}\right)^3 + {}^6C_4\left(\sqrt{\frac{x}{a}}\right)^2\left(-\sqrt{\frac{a}{x}}\right)^4 + {}^6C_5\left(\sqrt{\frac{x}{a}}\right)\left(-\sqrt{\frac{a}{x}}\right)^5$$

$$+ {}^6C_6\left(-\sqrt{\frac{a}{x}}\right)^6$$

$$= \frac{x^3}{a^3}+6\frac{x^2}{a^2}\left(\frac{\sqrt{x}}{\sqrt{a}}\right)\times\left(\frac{-\sqrt{a}}{\sqrt{x}}\right)+15\frac{x^2}{a^2}\times\frac{a}{x}+20\frac{x\sqrt{x}}{a\sqrt{a}}\times\left(\frac{-a\sqrt{a}}{x\sqrt{x}}\right)$$

S25.

$$(a + b)^4 - (a - b)^4 = [{}^4C_0\,a^4 + {}^4C_1\,a^3b + {}^4C_2\,a^2b^2 + {}^4C_3\,ab^3 + {}^4C_4\,b^4]$$
$$- [{}^4C_0\,a^4 - {}^4C_1\,a^3b + {}^4C_2\,a^2b^2 - {}^4C_3\,ab^3 + {}^4C_4\,b^4]$$
$$= 2 \times {}^4C_1\,a^3b + 2 \times {}^4C_3\,ab^3$$
$$= 2\,[4a^3b + 4ab^3]$$
$$= 8ab\,[a^2 + b^2]$$

Thus, $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4 = 8\sqrt{3}\cdot\sqrt{2}[3+2]$

$$= 8\sqrt{6}(5) = 40\sqrt{6}.$$

S26.

$$(a^2+\sqrt{a^2-1})^4+(a^2-\sqrt{a^2-1})^4 = [{}^4C_0(a^2)^4 + {}^4C_1(a^2)^3(\sqrt{a^2-1}) + {}^4C_2(a^2)^2(\sqrt{a^2-1})^2$$
$$+ {}^4C_3(a^2)^1(\sqrt{a^2-1})^3 + {}^4C_4(\sqrt{a^2-1})^4]$$
$$+ [{}^4C_0(a^2)^4 - {}^4C_1(a^2)^3(\sqrt{a^2-1}) + {}^4C_2(a^2)^2(\sqrt{a^2-1})^2$$
$$- {}^4C_3(a^2)^1(\sqrt{a^2-1})^3 + {}^4C_4(a^2)^0(\sqrt{a^2-1})^4]$$
$$= 2[{}^4C_0(a^2)^4 + {}^4C_2(a^2)^2(\sqrt{a^2-1})^2 + {}^4C_4(\sqrt{a^2-1})^4]$$

(Because other terms cancel)

$$= 2[a^8+6a^4(\sqrt{a^2-1})^2+(\sqrt{a^2-1})^4]$$
$$= 2\,[a^8 + 6a^4\,(a^2-1) + (a^2-1)^2]$$
$$= 2\,[a^8 + 6a^6 - 6a^4 + a^4 - 2a^2 + 1]$$
$$= 2\,[a^8 + 6a^6 - 5a^4 - 2a^2 + 1].$$

S27.

$$(x + 1)^6 + (x - 1)^6 = [{}^6C_0\,x^6 + {}^6C_1\,x^5 + {}^6C_2\,x^4 + {}^6C_3\,x^3 + {}^6C_4\,x^2 + {}^6C_5\,x^1 + {}^6C_6]$$
$$+ [{}^6C_0\,x^6 - {}^6C_1\,x^5 + {}^6C_2\,x^4 - {}^6C_3\,x^3 + {}^6C_4\,x^2 - {}^6C_5\,x^1 + {}^6C_6]$$

$$= 2\ [{}^6C_0\, x^6 + {}^6C_2\, x^4 + {}^6C_4\, x^2 + {}^6C_6]$$
$$= 2\,[x^6 + 15x^4 + 15x^2 + 1]$$

Thus, $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6 = 2[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15\,(\sqrt{2})^2 + 1]$

$$= 2\,[8 + 15\,(4) + 15\,(2) + 1]$$
$$= 2\ [8 + 60 + 30 + 1] = 198.$$

S28. By using Binomial Theorem, we have

$$(2x-3)^6 = [2x + (-3)]^6$$
$$= {}^6C_0\,(2x)^6\,(-3)^0 + {}^6C_1\,(2x)^5\,(-3)^1 + {}^6C_2\,(2x)^4\,(-3)^2 + {}^6C_3\,(2x)^3\,(-3)^3$$
$$+ {}^6C_4\,(2x)^2\,(-3)^4 + {}^6C_5\,(2x)^1\,(-3)^5 + {}^6C_6\,(2x)^0\,(-3)^6$$
$$= 1\,(2x)^6 + 6\,(2x)^5\,(-3) + 15\,(2x)^4\,(-3)^2 + 20\,(2x)^3\,(-3)^3 + 15\,(2x)^2\,(-3)^4$$
$$+ 6\ (2x)\,(-3)^5 + (-3)^6$$
$$= 64x^6 - 57x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729.$$

S29. By using Binomial Theorem, we have

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6\left(\frac{1}{x}\right)^0 + {}^6C_1(x)^5\left(\frac{1}{x}\right)^1 + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3$$
$$+ {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)^1\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$
$$= x^6 + 6x^5\left(\frac{1}{x}\right) + 15x^4\,\frac{1}{x^2} + 20x^3\cdot\frac{1}{x^3} + 15x^2\cdot\frac{1}{x^4} + 6x\cdot\frac{1}{x^5} + \frac{1}{x^6}$$
$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}.$$

S30. By using Binomial Theorem, we have

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5\left(\frac{1}{x}\right)^0 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3$$
$$+ {}^5C_4\left(\frac{x}{3}\right)^1\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{x}{3}\right)^0\left(\frac{1}{x}\right)^5$$
$$= \left(\frac{x}{3}\right)^5 + 5\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + 10\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + 10\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + 5\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^5$$
$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$$

S31. $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}\,b + {}^nC_2a^{n-2}\,b^2 + \ldots\ldots.. + {}^nC_nb^n$

$$(1+2x-3x^2)^5 = {}^5C_0(1+2x)^5 + {}^5C_1\,(1+2x)^4(-3x^2) + {}^5C_2\,(1+2x)^3\,(-3x^2)^2$$
$$+ {}^5C_3\,(1+2x)^2\,(-3x^2)^3 + {}^5C_4(1+2x)(-3x^2)^4 + {}^5C_5(-3x^2)^5 \qquad \ldots \text{(i)}$$

We know that,

$$(1 + 2x)^2 = 1 + 4x^2 + 4x,$$

$$(1 + 2x)^3 = 1 + 3(2x) + 3\,(2x)^2 + (2x)^3$$

$$(1 + 2x)^3 = 1 + 6x + 12x^2 + 8x^3$$

$$(1 + 2x)^4 = 1 + {}^4C_1\,2x + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4 \qquad \left\{\because {}^nC_r = \frac{n!}{r!(n-r)!}\right\}$$

$$= 1 + 8x + 24x^2 + 32x^3 + 16x^4$$

$$(1 + 2x)^5 = 1 + {}^5C_1(2x) + {}^5C_2(2x)^2 + {}^5C_3\,(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5$$

Substitute these values in equation (i), we get

$$(1 + 2x - 3x^2)^5 = 1(1 + 10x + 40x^2 + 15x^3 + 80x^4 + 32\,x^5) - 15x^2$$
$$(1 + 8x + 24x^2 + 32x^3 + 16x^4)(-3x^2) + 10 \times 9x^4\,(1 + 6x + 12x^2 + 8x^3)$$
$$+ 10(1 + 4x^2 + 4x)\,(-27\,x^6) + 5\,(1 + 2x)(81x^8) - 243x^{10}$$

$$(1 + 2x - 3x^2)^5 = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32\,x^5 - 15x^2$$
$$(1 + 8x + 24x^2 + 32x^3 + 16x^4) + 90x^4\,(1 + 6x + 12x^2 + 8x^3)$$
$$- 270x^6\,(1 + 4x^2 + 4x) + 405x^8\,(1 + 2x) - 243\,x^{10}$$

$$= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32\,x^5 - 15x^2 - 120x^3 - 360x^4 - 480x^5$$
$$- 240x^6 + 90x^4 + 540x^5 + 1080x^6 + 720\,x^7 - 270x^6 - 1080x^8$$
$$- 1080x^7 + 405x^8 + 810x^9 - 243\,x^{10}$$

$$(1 + 2x - 3x^2)^5 = 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7$$
$$- 675x^8 + 810x^9 - 243x^{10}$$

Date: 10/10/2021

Q1. Find the general term in the expansion of $(x^2 - y^2)^6$.

Q2. Find the general term in the expansion $(1 - x^2)^{12}$.

Q3. Find the general term in the expansion of $\left(2x + \frac{1}{x}\right)^5$.

Q4. Find the $(n + 1)^{th}$ term from the end in the expansion of $\left(x - \frac{1}{x}\right)^{3n}$.

Q5. Find the 5^{th} term from the end in the expansion of $\left(\frac{x}{2} + \frac{1}{x}\right)^{10}$.

Q6. Find the 4^{th} term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$.

Q7. The coefficient of the 8^{th} term in the expansion of $(1 + x)^{10}$ is

Q8. Write the general term in the expansion of : $(x^2 - yx)^{12}$, $x \neq 0$.

Q9. Find the 4^{th} term in the expansion of $(x - 2y)^{12}$.

Q10. Find the 4^{th} term in the expansion of $(x - 2y)^{12}$.

Q11. Find the r^{th} term form the end in $(x + a)^n$.

Q12. Find the fifth term in the expansion of $(2a + 2b)^{12}$ and hence evaluate it when $a = \frac{1}{3}$ and $b = \frac{1}{4}$.

Q13. Find the 12^{th} term in the expansion of $\left(x + \frac{1}{x}\right)^{15}$.

Q14. Find the 10^{th} term in the expansion of $\left(2x^2 + \frac{1}{x}\right)^{12}$.

Q15. Find the sixth term in the expansion of $\left(2x - \frac{1}{x^2}\right)^7$.

Q16. Find the 6^{th} term in the expansion of $\left\{2x^2 - \frac{1}{3x^2}\right\}^{10}$.

Q17. Find the 5^{th} term from the end in the expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$

Q18. Find the 13^{th} term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$.

Q19. Find a if the 17^{th} and 18^{th} terms of the expansion $(2 + a)^{50}$ are equal.

Date: 10/10/2021

S1. $\because$ General term in the expansion of $(a-b)^n$ is $T_r = {}^nC_r\, a^{n-r}\, b^r\,.\,(-1)^r$.

We have $(x^2 - y^2)^6$

$$T_r = {}^nC_r\, a^{n-r}\, b^r\,.\,(-1)^r.$$

$$\therefore \quad T_r = {}^6C_r (x^2)^{6-r}\,(y^2)^6.(-1)^r$$

$$T_r = {}^6C_r\,(x^2)^{6-r}\,y^{12}\,.\,(-1)^r$$

S2. $\because$ General term in the expansion of $(a-b)^n$ is $T_r = {}^nC_r\, a^{n-r}\, b^r\,.\,(-1)^r$.

$$(1 - x^2)^{12} = (1 + (-x^2))^{12}$$

$$\therefore \quad T_{r+1} = {}^{12}C_r (1)^{12-r}(-x^2)^r$$

$$= (-1)^r.{}^{12}C_r\, x^{2r}$$

S3. $\because$ General term in the expansion of $(a+b)^n$ is $T_r = {}^nC_r\, a^{n-r}\, b^r$.

General term $\quad T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

$$T_{r+1} = {}^5C_r\,(2x)^{5-r}\left(\frac{1}{x}\right)^r, \qquad \left[T_{r+1} = {}^5C_r\, 2^{5-r}\, x^{5-r}\, \frac{1}{x^r}\right]$$

$$T_{r+1} = {}^5C_r\, 2^{5-r}\, x^{5-2r}$$

S4. $(n+1)^{th}$ term from the end = $(3n - n - 1 + 2) = (2n+1)^{th}$ term from the beginning

$$T_{r+1} = (-1)^r\, {}^nC_r\, a^{n-r}\, b^r$$

$$T_{2n+1} = (-1)^{2n}\,.\,{}^{3n}C_{2n}\,(x)^{3n-2n}\left(\frac{1}{x}\right)^{2n} = \frac{(3n)!}{(2n)!\,n!}\,x^n\cdot\frac{1}{x^{2n}} = \frac{(3n)!}{(2n)!\,n!}\cdot\frac{1}{x^n}$$

S5.

$$T_{r+1} = {}^{10}C_r\left(\frac{x}{2}\right)^{10-r}\left(\frac{2}{x}\right)^r$$

Now, p^{th} term from the end is same as $(n+2-p)^{th}$ term from the beginning. Here, 5^{th} term from the end is same as 7^{th} term from the beginning

$$\Rightarrow \quad r = 6$$

$$T_7 = 4.{}^{10}c_6\cdot x^{-2} = 840x^{-2}.$$

S6. 4^{th} term from the end is same as 7^{th} term from the beginning

$$\Rightarrow \quad r = 6$$

$$T_7 = {}^9C_6\left(\frac{x^3}{2}\right)^3\left(-\frac{2}{x^2}\right)^6 = 8\cdot{}^9c_6\,.\,x^{-3} = 672x^{-3}.$$

S7. The general term in the expansion of $(1 + x)^{10}$ is

$$T_{r+1} = {}^{10}C_r\, x^r$$

$$\therefore \quad t_8 = {}^{10}C_7\, x^7$$

And so, the coefficient of the 8th term in the expansion of

$$(1 + x)^{10} \text{ is } {}^{10}C_7 = {}^{10}C_3 = \frac{10 \times 9 \times 8}{3 \times 2} = 120.$$

S8. We have, T_{r-1} in $(x^2 - yx)^{12} = {}^{12}C_r\,(x^2)^{12-r}\,(-yx)^r$

$$= {}^{12}C_r\, x^{24-2r}\,(-1)^r\,(y)^r\,(x)^r$$

$$= {}^{12}C_r\, x^{24-r}\, y^r\,(-1)^r.$$

S9. 4th term in $(x - 2y)^{12} = T_4 = T_{3+1}$

$$= {}^{12}C_3\,(x)^{12-3}\,(-2y)^3 \qquad [T_{r-1} \text{ in } (a+b)^n = {}^nC_r a^{n-r} b^r]$$

$$= {}^{12}C_3\,(x)^9\,(-2)^3\,(y)^3$$

$$= \frac{12 \times 11 \times 10}{3 \times 2} \cdot (-8) \times x^9 y^3$$

$$= -1760 x^9 y^3.$$

S10. We know that $(r+1)^{th}$ term in the expansion of $(a + b)^n$ is given by

$$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r.$$

$\therefore$ In the expansion $(x - 2y)^{12}$.

$$T_4 = T_{3+1} = {}^{12}C_3(x)^{12-3}\,(-2y)^3$$

$$= \frac{12!}{3!(12-3)!} \times x^9 \times (-8y^3)$$

$$= \frac{-12 \times 11 \times 10 \times 9!}{3 \times 2 \times 1 \times 9!} x^9 \times 8y^3$$

$$= -2 \times 11 \times 10 \times 8 \times x^9 y^3 = -1760\, x^9 y^3$$

S11. There are $(n + 1)$ terms in the expansion.

r^{th} term from the end = $\{n + 1 - (r - 1)\}^{th}$ term from the beginning = $(n + 2 - r)^{th}$ term from the beginning.

Hence, $T_{n+2-r} = {}^nC_{n+1-r}\, x^{n-(n+1-r)}\, a^{n+1-r} = {}^nC_{n-r+1}\, x^{n-n+r-1}\, a^{n+1-r}$

$$= {}^nC_{n+1-r}\, x^{r-1} a^{n+1-r}.$$

S12. $T_{r+1} = {}^nC_r\, a^{n-r} b^r$, $\qquad T_5 = {}^{12}C_4\,(2a)^8 \cdot (3b)^4 \qquad [r = 4]$

$a = \frac{1}{3}, b = \frac{1}{4}$, $\qquad T_5 = \frac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 4 \times 3 \times 2} \times 2^8 \times \frac{1}{3^8} \times 3^4 \times \frac{1}{4^4}$

$$T_5 = \frac{11\times 5\times 9\times 2^8}{3^4\times 2^8} \qquad \Rightarrow T_5 = \frac{55}{9}$$

S13.

$$T_{r+1} = {}^{15}c_r x^{15-r}\left(-\frac{1}{x}\right)^r$$

for 12^{th} term we put $r = 11$

$$\Rightarrow \qquad T_{12} = {}^{15}c_{11}x^4\left(-\frac{1}{x}\right)^{11}$$

$$= {}^{15}c_{11}x^4\left(-\frac{1}{x}\right)^{11}$$

$$= -{}^{15}c_{11}x^{-7} = -1365x^{-7}.$$

S14. $\because$ General term in the expansion of $(a + b)^n$ is $T_r = {}^nC_r\, a^{n-r}\, b^r$.

$$T_{r+1} = {}^{12}C_r(2x^2)^{12-r}\left(\frac{1}{x}\right)^r$$

for 10^{th} term we put $r = 9$

$$T_{10} = {}^{12}c_9(2x^2)^3\left(\frac{1}{x}\right)^9$$

$$= 2^3\cdot{}^{12}c_9x^{-3} = 1760\ x^{-3}.$$

S15. $\because$ General term in the expansion of $(a - b)^n$ is $T_r = (-1)^r\, {}^nC_r\, a^{n-r}\, b^r$.

$T_{r+1} = (-1)^r\, {}^nC_r\, a^{n-r}\, b^r,$ $\qquad T_6 = (-1)^5 . {}^7C_5\ (2x)^2\ \left(\frac{1}{x^2}\right)^5$

$$T_6 = -\frac{7\times 6\times 5!}{5!\times 2}\times 4x^2\times\frac{1}{x^{10}} \Rightarrow \quad T_6 = \frac{-84}{x^8}.$$

S16. The general term in the expansion $\left\{2x^2 - \frac{1}{3x^2}\right\}^{10}$

$$t_{r+1} = {}^{10}C_r(-1)^r\cdot\ (2x^2)^{10-r}\left(\frac{1}{3x^2}\right)^r \qquad \ldots \text{(i)}$$

$$= {}^{10}C_r\,(-1)^r\,(2)^{10-r}\,(3)^{-r}\ x^{20-4r}$$

Putting $r = 5$ in (i) we get:

$$t_6 = {}^{10}C_5(-1)^5\,(2)^5\,(3)^{-5}\ x^0$$

$$= \frac{10\times 9\times 8\times 7\times 6}{5\times 4\times 3\times 2\times 1}\times(-1)\times\frac{32}{243} = -\frac{896}{27}.$$

S17. We know that p^{th} term from the end

$$= (n - p + 2)^{th} \text{ term from the beginning}$$

$\therefore$ In the expansion $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$.

We have

5^{th} term from the end

$$= (9 - 5 + 2)^{th} \text{ term} = 6^{th} \text{ term from the end}$$

$$t_6 = t_{5+1} = (-1)^5 \cdot {}^9C_5 \left(\frac{x^3}{2}\right)^{9-5}\left(\frac{2}{x^2}\right)^5$$

$$= -\,{}^9C_4 \left(\frac{x^3}{2}\right)^4\left(\frac{2}{x^2}\right)^5$$

$$= -\left(\frac{9\times 8\times 7\times 6}{4\times 3\times 2\times 1}\times\frac{x^{12}}{16}\cdot\frac{32}{x^{10}}\right)$$

$$= -252\,x^2.$$

S18. 13^{th} term in $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18} = T_{13} = T_{12+1}$

$$= {}^{18}C_{12}\,(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= {}^{18}C_{12}\,(9x)^6\left(\frac{-1}{3}\right)^{12}\left(\frac{1}{\sqrt{x}}\right)^{12}$$

$$= {}^{18}C_{12}\,(9x)^6\left(\frac{-1}{3}\right)^{12}\left(x^{\frac{1}{2}}\right)^{-12}$$

$$= {}^{18}C_{12}\,(9x)^6\left(\frac{-1}{3}\right)^{12}(x)^{-6}$$

$$= {}^{18}C_{12}\,(3^2)^6\,(x)^6\,(-1)^{12}\,(3)^{-12}\,(x)^{-6}$$

$$= {}^{18}C_{12}\,(3^{12})\,(x^6)\,(3)^{-12}\,(x)^{-6}$$

$$= {}^{18}C_{12} = 18564.$$

S19. The $(r + 1)^{th}$ term of the expansion $(x + y)^n$ is given by $T_{r+1} = {}^nC_r\,x^{n-r}\,y^r$. For the 17^{th} term, we have, $r + 1 = 17$, *i.e.*, $r = 16$

Therefore, $T_{17} = T_{16+1} = {}^{50}C_{16}\,(2)^{50-16}\,a^{16}$

$$= {}^{50}C_{16}\,2^{34}\,a^{16}$$

Similarly, $T_{18} = {}^{50}C_{17}\, 2^{33}\, a^{17}$

Given that $T_{17} = T_{18}$

So, ${}^{50}C_{16}\, (2)^{34}\, a^{16} = {}^{50}C_{17}\, (2)^{33}\, a^{17}$

Therefore, $$\frac{{}^{50}C_{16}\, .\, 2^{34}}{{}^{50}C_{17}\, .\, 2^{33}} = \frac{a^{17}}{a^{16}}$$

i.e., $$a = \frac{{}^{50}C_{16} \times 2}{{}^{50}C_{17}} = \frac{50!}{16!34!} \times \frac{17!\,.\,33!}{50!} \times 2 = 1.$$

Date: 10/10/2021

Q1. If $n = 2m$ then find the middle term in the expansion of $(x + a)^n$.

Q2. Find the middle term (s) in the expansion of $(2x + 3y)^9$

Q3. Find the middle term in the expansion of $\left(x - \frac{1}{2y}\right)^{10}$.

Q4. Find the middle term in the expansion of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^6$.

Q5. Find the middle term in the expansion of $(x - a)^8$.

Q6. Find the middle term (s) in the expansion of

$$\left(x + \frac{1}{x}\right)^{10}$$

Q7. Find the middle term (s) in the expansion of $\left(\frac{a}{3} + 2b\right)^8$

Q8. Find the middle term (s) in the expansion of $(1 + x)^{2n}$

Q9. Find the middle term in the expansion of $\left(3x - \frac{x^3}{6}\right)^7$.

Q10. Find the middle term in the expansion of $\left(x - \frac{1}{2x}\right)^{10}$.

Q11. Find the coefficient of middle term in the expansion of $(3x + 2)^4$.

Q12. Find the middle terms in the expansions of : $\left(\frac{x}{3} + 9y\right)^{10}$.

Q13. Find the middle terms in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$.

Q14. Find the middle terms in the expansion of $\left(3a - \frac{a^3}{6}\right)^9$.

Q15. Find the middle terms in the expansion of $\left(3 - \frac{x^3}{6}\right)^7$.

Q16. Find the middle term in the expansion of $\left(\frac{2x^2}{3} + \frac{3}{2x^2}\right)^{10}$.

Q17. Find the middle term in the expansion of $\left(x - \frac{1}{2}y\right)^{10}$.

Date: 10/10/2021

S1. $(x + a)^n = (x + a)^{2m}$ $[\because\ n = 2m]$.

Now, the index $2m$ is even and so the middle term in the expansion of $(x + a)^{2m}$ is $\left(\frac{2m}{2}+1\right)^{th}$ term *i.e.*, $(m + 1)^{th}$ term.

S2.
$$T_{r+1} = {}^9c_r\,(2x)^{9-r}\,(3y)^r$$

Here, the total number of terms in the expansion will be 10, therefore, T_5 and T_6 will be middle terms $\Rightarrow$ $r = 4, 5$

$$T_5 = {}^9c_4(2x)^5\,(3y)^4 = 326592x^5y^4$$
$$T_6 = {}^9c_5(2x)^4\,(3y)^5 = 489888x^4y^5.$$

S3. Since total no. of terms in the given expression is 11.

$\therefore$ middle term = 6th term

$$t_6 = t_{5+1}$$

$$= (-1)^5 \cdot {}^{10}C_5 \cdot x^{10-5}\left(\frac{1}{2y}\right)^5$$

$$= -63\frac{x^5}{8y^5}.$$

S4. The general term in the expansion of $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^6$ is

$$t_{r+1} = {}^6C_r\,(-1)^r\,(\sqrt{x}\,)^{6-r}\left(\frac{1}{\sqrt{x}}\right)^r = {}^6C_r\,(-1)^r\,(x)^{3-r} \qquad \text{... (i)}$$

Now, the index being 6, which is even, the middle term of the given expansion is $\left(\frac{6}{2}+1\right)$ th term *i.e.*, 4^{th} term.

Hence middle term is $t_4 = {}^6C_4\,(-1)^4\,x^{-1} = {}^6C_4\ x^{-1} = 15x^{-1}$

S5. The index of $(x - a)^8$ is 8 which is even and so the middle term in the expansion of $(x - a)^8$ is $\left(\frac{8}{2}+1\right)$ th term *i.e.*, 5^{th} term.

Now, the general term in the expansion of $(x - a)^8$ is

$$t_{r+1} = {}^8C_r\,(-1)^r\,x^{8-r}\,a^r \qquad \text{... (i)}$$

Putting $r = 4$ in Eq. (i) we get

$$t_5 = {}^8C_4\,(-1)^4\,x^4\,a^4 = {}^8C_4\,x^4\,a^4.$$

S6.

$$T_{r+1} = {}^{10}c_r \cdot x^{10-r}\left(\frac{1}{x}\right)^r$$

Here, the total number of terms in the expansion will be 11 therefore, T_6 will be the middle term.

$$T_6 = {}^{10}c_5 x_5 \left(\frac{1}{x}\right)^5 = {}^{10}c_5 = 252$$

S7.

$$T_{r+1} = {}^{8}C_r\left(\frac{a}{3}\right)^{8-r}(2b)^r$$

Here, the total number of terms in the expansion will be 9, therefore, T_5 will be the middle term

$\Rightarrow$ $r = 4$

$$T_5 = {}^{8}C_4\left(\frac{a}{3}\right)^4(2b)^4 = \frac{1120}{81}(ab)^4$$

S8. $T_{r+1} = {}^{2n}c_r x^r$

Here, total number of terms in the expansion will be $2n + 1$, therefore, T_{n+1} will be the middle term

$$T_{n+1} = {}^{2n}C_n x^n = \frac{(2n)!}{(n!)^2}x^n$$

S9. Since the given expression contains 8 terms.

$\therefore$ middle term are 4th and 5th terms

Now $t_4 = t_{3+1} = (-1)^3 \times {}^7C_3(3x)^{7-3}\left(\frac{x^3}{6}\right)^3$

$$= \left((-35)\times 81\times x^4\times\frac{x^9}{216}\right)$$

$$= -\frac{105x^{13}}{8}$$

and $t_5 = t_{4+1} = (-1)^4 . {}^7C_4(3x)^{7-4}\left(\frac{x^3}{6}\right)^4$

$$= \left(35\times 27\times x^3\times\frac{x^{12}}{1296}\right) = \frac{35x^{15}}{48}$$

Hence the middle terms are $-\frac{105\,x^{13}}{8}$ and $\frac{35x^{15}}{48}$.

S10. The general term in the expansion of $\left(x - \frac{1}{2x}\right)^{10}$ is

$$t_{r+1} = {}^{10}C_r (-1)^r x^{10-r} \left(\frac{1}{2x}\right)^r$$

$$= {}^{10}C_r (-1)^r 2^{-r} x^{10-2r} \quad \text{... (i)}$$

Now, the index being 10, the middle term in $\left(\frac{10}{2}+1\right)$th term *i.e.*, 6th term.

$\left[\because \text{ When index } n \text{ is even, then middle is } \left(\frac{n}{2}+1\right) \text{th term}\right]$

$\therefore$ The middle term is $t_6 = {}^{10}C_5 (-1)^5 2^{-5} x^0$ [Putting $r = 5$ in Eq. (i)]

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \times (-1) \times \frac{1}{32} = \frac{-63}{8}$$

S11. The index of $(3x + 2)^4$ is 4 which is even and so the middle term in the expansion of $(3x + 2)^4$ is $\left(\frac{4}{2}+1\right)$th term *i.e.*, 3rd term.

Now, the general term in the expansion of $(3x + 2)^4$ is

$$t_{r+1} = {}^4C_r (3x)^{4-r} (2)^r \quad \text{... (i)}$$

Putting $r = 2$ in Eq. (i), we get

$$t_3 = {}^4C_2 (3x)^2 (2)^2$$

$$= \frac{4 \times 3}{2 \times 1} \times 9x^2 \times 4 = \mathbf{216\,x^2}.$$

$\therefore$ The coefficient of the middle term is 216.

S12. The index of $\left(\frac{x}{3} + 9y\right)^{10}$ is 10, which is an even natural number.

Hence, Middle term = $T_{\frac{10+2}{2}} = T_6 = T_{5+1}$

$$\therefore \quad T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5$$

$$= {}^{10}C_5 \left(\frac{x}{3}\right)^5 (9y)^5$$

$= {}^{10}C_5 (x)^5 \left(\frac{1}{3}\right)^5 (9)^5 (y)^5$

$= {}^{10}C_5 (3)^{-5} (3^2)^5 (x)^5 (y)^5$

$= {}^{10}C_5 (3)^{-5} (3)^{10} x^5 y^5$

$= {}^{10}C_5 3^5 x^5 y^5 = (252)(243) x^5 y^5$

$= 61236 x^5 y^5.$

S13. $n = 15$ (odd)

Middle term = $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ term, i.e., 8^{th} and 9^{th} term

$T_{r+1} = (-1)^r . {}^{15}C_r (3x)^{15-r} \left(\frac{2}{x^2}\right)^r$

$T_{r+1} = (-1)^r . {}^{15}C_r (3)^{15-r} 2^r x^{15-r-2r}$

$T_{r+1} = (-1)^r . {}^{15}C_r (3)^{15-r} 2^r x^{15-3r}$

$T_8 = (-1)^7 \, {}^{15}C_r (3)^8 2^7 x^{-6}$

$= -\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8!}{8! 7 \times 6 \times 5 \times 4 \times 3 \times 2} 3^8 2^7 \frac{1}{x^6}$

$= -6435 \times 3^8 \times 2^7 \times \frac{1}{x^6}$

$T_9 = (-1)^8 \, {}^{15}C_8 3^7 2^8 x^{15-24} = {}^{15}C_7 3^7 . 2^8 x^{-9}$ $\quad [\because {}^{15}C_8 = {}^{15}C_7]$

$T_9 = \frac{6435 \times 3^7 \times 2^8 \times 1}{x^9}$, $\quad T_9 = \frac{6435 \times 3^7 \times 2^8}{x^9}$.

S14. Here n = odd

Middle term = $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ term, *i.e.*, 5^{th} and 6^{th} term

$T_{r+1} = {}^nC_r a^{n-r} b^r$,

$T_{r+1} = (-1)^r \, {}^9C_r (3a)^{9-r} \left(\frac{a^3}{6}\right)^r$

$= (-1)^r \, {}^9C_r \, 3^{9-r} a^{9-r} \frac{a^{3r}}{6^r}$, $\quad T_{r+1} = (-1)^r \frac{{}^9C_r 3^{9-r} a^{9+2r}}{6^r}$

$T_5 = (-1)^4 \, {}^9C_4 (3)^5 \frac{a^{17}}{6^4} = {}^9C_4 \frac{3^5}{3^4} \frac{a^{17}}{2^4} = {}^9C_4 \frac{3}{16} a^{17}$

$T_5 = \frac{9 \times 5 \times 7 \times 6 \times 5!}{5! \times 4 \times 3 \times 2} \times \frac{3}{16} a^{17} = \frac{189}{8} a^{17}$

$$T_6 = \frac{(-1)^5 \ {}^9C_5 \ 3^4 \ a^{19}}{6^5}, \qquad T_6 = \frac{9\times8\times7\times6\times5!}{5!\times4\times3\times2}\frac{3^4}{3^5.2^5}a^{19}$$

$$T_6 = -\frac{6\times7}{32}a^{19} \quad \Rightarrow \quad T_6 = -\frac{21}{16}a^{19}.$$

S15.

The given expression = $\left(3-\frac{x^3}{6}\right)^7$. Here $n = 7$, which is an odd number.

So, $\left(\frac{7+1}{2}\right)^{\text{th}}$, and $\left(\frac{7+1}{2}+1\right)^{\text{th}}$ *i.e.*, 4th and 5th terms are two middle terms.

Now, $$T_4 = T_{3+1} = {}^7C_3(3)^{7-3}\left(-\frac{x^3}{6}\right)^3 = (-1)^3.\ {}^7C_3(3)^4\left(\frac{x^3}{6}\right)^3$$

$$= -35\times81\times\left(\frac{x^3}{6}\right)^3 = -\frac{105\,x^9}{8}$$

$$T_5 = T_{4+1} = {}^7C_4(3)^{7-4}\left(-\frac{x^3}{6}\right)^4 = {}^7C_4(3)^3\left(-\frac{x^3}{6}\right)^4$$

$$= 35\times27\times\frac{x^{12}}{1296} = \frac{35x^{12}}{48}$$

Hence, the middle terms are $\frac{-105x^9}{8}$ and $\frac{35x^{12}}{48}$.

S16.

The general term in the expansion of $\left(\frac{2x^2}{3}+\frac{3}{2x^2}\right)^{10}$ is

$$t_{r+1} = {}^{10}C_r\left(\frac{2x^2}{3}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r(2)^{10-2r}(3)^{2r-10}(x)^{20-4r} \qquad \ldots \text{(i)}$$

Now, the index is 10, which is even, so the middle term of the expansion is $\left(\frac{10}{2}+1\right)$th *i.e.*, the 6th term.

Putting $r = 5$ in Eq. (i), we get

$$t_6 = {}^{10}C_5 (2)^0 (3)^0$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = \mathbf{252}.$$

$\therefore$ The middle term in the given expansion is 252.

S17.

The general term in the expansion of $\left(x - \frac{1}{2}y\right)^{10}$ is

$$t_{r+1} = {}^{10}C_r (-1)^r x^{10-r} \left(\frac{y}{2}\right)^r \qquad \dots \text{(i)}$$

Now, the index is 10 which is even and so the middle term in the expansion is the $\left(\frac{10}{2} + 1\right)$th *i.e.*, the 6th term.

Putting $r = 5$ in Eq. (i), we get

$$t_6 = {}^{10}C_5 (-1)^5 \cdot x^5 \left(\frac{y}{2}\right)^5$$

$$= (-1) \times \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \times \frac{x^5 y^5}{32}$$

$$= \frac{-63 x^5 y^5}{8}.$$

Date: 10/10/2021

Q1. Find the coefficient of x^6 in the expansion of $\left(3x^2 - \frac{1}{3x}\right)^9$.

Q2. Find the coefficient of x^7 in the expansion of $\left(x - \frac{1}{x}\right)^{13}$.

Q3. Find the coefficient of x^7 in the expansion of $\left(x^2 + \frac{1}{x}\right)^{11}$.

Q4. Find the coefficient of x^4 in the expansion of $(1 + x)^{-2}$, where $|x| < 1$.

Q5. Find the coefficient of x^r in the expansion of $(1 + x)^{-2}$, where $|x| < 1$.

Q6. Find the coefficient of x in the expansion of $(1 + x + x^2 + x^3)^{-3}$.

Q7. Find the coefficient of x^5 in the expansion of $(1 + 2x + 3x^2 + ...)^{-3/2}$.

Q8. If in the expansion of $(1 + x)^{21}$, the coefficients of x^r and x^{r+1} are equal, then find the value of r.

Q9. Find the coefficient of : x^5 in $(x + 3)^8$.

Q10. Find the coefficient of x^6y^3 in the expansion of $(x + 2y)^9$.

Q11. If the coefficients of $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms of the expansion $(1 + x)^{34}$ are equal, find r.

Q12. Find the coefficient of x^2 in the expansion of $\left(3x - \frac{1}{x}\right)^6$.

Q13. Find the coefficient of y^9 in the expansion of $(5 - 2y)^{11}$.

Q14. Find the coefficient of x^{14} in the expansion of $(1 + x + x^2 + x^3)^6$.

Q15. Find the coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$.

Q16. Find the coefficient of x^4 in the expansion of $\left(\frac{1-x}{1+x}\right)^2$.

Q17. Find the coefficient of x^{-n} in $(1 + x)^n \left(1 + \frac{1}{x}\right)^n$.

Q18. Find the coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$.

Q19. Find the coefficient of x^5 in the expansion of $(1 + x^2)^5(1 + x)^4$.

Q20. Find the coefficient of x^{24} in the expansion of $(1 + x^2)^{12}(1 + x^{12})(1 + x^{24})$.

Q21. Find the coefficient of x in the expansion of $\left(x^2 + \frac{a}{x}\right)^5$.

Q22. If the r^{th} term in the expansion of $\left(\frac{x}{3} - \frac{2}{x^2}\right)^{10}$ contains x^4, then find the value of r.

Q23. In the expansion of $\left(x^2 + \frac{1}{x}\right)^n$, the coefficient of the fourth term is equal to the coefficient of the ninth term. Find n and the sixth term of expansion.

Q24. Find the coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$.

Q25. If the coefficient of r^{th} term and $(r + 1)^{th}$ term in the expansion of $(1 + x)^{20}$ are in the ratio 1 : 2, then find the value of r.

Q26. Find the coefficient of x^{32} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

Q27. Find the coefficient of x^5 in the binomial expansion of $(x + 3)^8$.

Q28. Find the coefficient of $a^5 b^7$ in $(a - 2b)^{12}$.

Q29. Find the coefficient of x^5 in the expansion of the product $(1 + 2x)^6 (1 - x)^7$.

Q30. Prove that coefficient of x^n in $(1 + x)^{2n}$ is twice the coefficient of x^n in $(1 + x)^{2n-1}$.

Q31. Find n if the coefficient of the 4^{th} and 13^{th} terms in the expansion of $(a + b)^n$ are equal.

Q32. Show that the coefficient of the middle term in the expansion of $(1 + a)^8$ is equal to the sum of the coefficients of middle term in the expansion of $(1 + a)^7$.

Q33. If the coefficient of the second, third and fourth terms in the expansion of $(1 + x)^{2n}$ are in A.P., then show that $n = \frac{7}{2}$.

Q34. If the consecutive coefficients in the expansion of $(1 + x)^n$ are in the ratio 6 : 33 : 110, find n.

Q35. Find the coefficient of x^{-9} in the expansion of $\left(\frac{x^2}{2} - \frac{2}{x}\right)^9$.

Q36. Find the coefficient of x^4 in the expansion of $(1 + x + x^3 + x^4)^{10}$.

Q37. Find the coefficient of x^2 term in the binomial expansion of $\left(\frac{1}{3}x^{1/2} + x^{-1/4}\right)^{10}$.

Q38. Find the coefficient of x^{-11} in the expansion of $\left(\sqrt{x} - \frac{2}{x}\right)^{17}$.

Q39. Find the coefficient of x^{11} in the expansion of $(1 + 3x + 2x^2)^6$.

Q40. Find the Coefficient of x^5 in $(1 + 2x + 3x^2 + ...)^{3/2}$.

Q41. Find the coefficient of x^3 in the expansion of $(1 + x + x^2)^n$.

Q42. Find the coefficient of x^{10} in the expansion of $(1 + x^2 - x^3)^8$.

Q43. If $0 \le r \le n$, then find the coefficient of x^r in the expansion of $P = 1 + (1 + x) + (1 + x)^2 + ... + (1 + x)^n$.

Q44. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ be same, then find the value of 'a'.

Q45. Find the coefficient of x^3 in the expansion of $\left(\sqrt{x^5} + \frac{3}{\sqrt{x^3}}\right)^6$.

Q46. Find the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$.

Q47. Find the coefficient of $a^3b^4c^2$ in the expansion of $(1 + a + b - c)^{10}$.

Q48. Find the coefficient of x^6 in the expansion of $(1 + x)^6 + (1 + x)^7 + ... + (1 + x)^{15}$.

Q49. If in the expansion of $(1 + x)^n$, the coefficient of p^{th} and $(p + 1)^{th}$ terms are respectively p and q, then prove that $p + q = n + 1$.

Q50. Find the coefficient of x^5 in the expansion of $(1 + x)^{21} + (1 + x)^{22} + ... + (1 + x)^{30}$.

Q51. If the coefficients of $(m + 1)^{th}$ term and $(m + 3)^{th}$ term in expansion of $(1 + x)^{20}$ are equal, then find the value of m.

Q52. Show that the coefficient of the middle term of $(1 + x)^{2n}$ is equal to the sum of the coefficients of the two middle terms of $(1 + x)^{2n-1}$.

Q53. Find the coefficient of $a^5b^6c^7$ in the expansion of $(bc + ca + ab)^9$.

Q54. Find the coefficient of $a^4b^3c^2d$ in the expansion of $(a - b + c - d)^{10}$.

Q55. Find coefficient of x^n in the expansion of $(1 - 4x)^{-1/2}$.

Q56. The sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^3}\right)^m$, $x \neq 0$, m being a natural number is 559. Find the term of the expansion containing x^3.

Q57. Find the coefficient of a^4 in the product $(1 + 2a)^4(2 - a)^5$ using Binomial Theorem.

Q58. If the coefficients of x, x^2 and x^3 in the binomial expansion of $(1 + x)^{2n}$ are in arithmetic progression, then prove that $2n^2 - 9n + 7 = 0$.

Q59. If in the binomial expansion of $(1 + x)^n$, the coefficients of the fifth, sixth and seventh terms are in A.P. Find all values of n for which this can happen.

Q60. If the coefficients of $(r - 1)^{th}$, r^{th} and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio 1 : 3 : 5. Find both n and r.

Q61. If the coefficient of three consecutive terms in the expansion of $(1 + a)^n$ are in the ratio 1 : 7 : 42. Find n.

Q62. If the second, third and fourth terms in the expansion of $(x + a)^n$ are 240, 720 and 1080 respectively. Find n, x and a.

Q63. If the coefficients of x^7 and x^8 in $\left(2 + \frac{x}{3}\right)^n$ are equal, then find the value of n.

Q64. If in the expansion of $(1 + x)^n$, the coefficients of 2^{nd}, 3^{rd} and 4^{th} terms are in A.P. Then find the value of n.

Q65. Find the coefficient of x^7 in the expansion of $(1 + 3x - 2x^3)^{10}$.

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S1.
$$T_{r+1} = {}^9C_r(3x^2)^{9-r}\left(-\frac{1}{3x}\right)^r = (-1)^r\,{}^9C_r \times 3^{9-2r}\,x^{18-3r}$$

Now, $18 - 3r = 6 \quad \Rightarrow \quad r = 4$

$\Rightarrow$ $T_5 = (-1)^4 \cdot {}^9c_4 \cdot 3^1 \cdot x^6 = 378x^6$

$\Rightarrow$ The required coefficient is 378

S2.
$$T_{r+1} = {}^{13}C_r \cdot x^{13-r}\left(-\frac{1}{x}\right)^r = (-1)^r\,{}^{13}C_r \times x^{13-2r}$$

Now, $13 - 2r = 7 \quad \Rightarrow \quad r = 3$

$$T_4 = (-1)^3 \cdot {}^{13}C_3\,x^7 = -286x^7$$

$\Rightarrow$ The required coefficient is -286.

S3.
$$T_{r+1} = {}^{11}C_r(x^2)^{11-r}\left(\frac{1}{x}\right)^r = {}^{11}C_r\,x^{22-2r-r} = {}^{11}C_r x^{22-3r}$$

$$22 - 3r = 7 \quad \Rightarrow \quad r = 5, \qquad T_6 = {}^{11}C_5x^7$$

$$\text{Coefficient of } x^7 = {}^{11}C_5 = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6 \times 5 \times 4 \times 3 \times 2 \times 5!} = 462.$$

S4. By Binomial Expansion:

$$(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots$$

$\therefore$ The coefficient of x^4 in the expansion of $(1 + x)^{-2}$ is 5.

S5. By Binomial Expansion:

$$(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots + (-1)^r(r + 1)x^r + \ldots$$

$\therefore$ The coefficient of x^r in the expansion is $(-1)^r(r + 1)$.

S6. We have

$$(1 + x + x^2 + x^3)^{-3} = \{(1 + x) + x^2(1 + x)\}^{-3}$$
$$= \{(1 + x)(1 + x^2)\}^{-3} = (1 + x)^{-3}(1 + x^2)^{-3}$$
$$= \{1 - 3x + 6x^2 - 10x^3 + \ldots\} \times \{1 - 3x^2 + 6x^4 - 10x^6 + \ldots\}$$

$\therefore$ The coefficient of x is -3.

S7. We have,

$$(1 + 2x + 3x^2 + \ldots)^{-3/2} = \{(1 - x)^{-2}\}^{-3/2} \qquad [\because\ (1 - x)^{-2} = 1 + 2x + 3x^2 + \ldots]$$

$$= (1 - x)^3 = {}^3C_0 - {}^3C_1 x + {}^3C_2 x^2 - {}^3C_3 x^3$$

There is no term containing x^5. And so, the coefficient of x^5 is 0.

S8. By Binomial Expansion:

$$(1 + x)^{21} = {}^{21}C_0 + {}^{21}C_1 x + {}^{21}C_2 x^2 + \ldots + {}^{21}C_{21} x^{21} \qquad [\because\ {}^nC_r = {}^nC_{n-r}]$$

$\therefore$ The coefficient of x^r and x^{r+1} are ${}^{21}C_r$ and ${}^{21}C_{r+1}$ respectively.

Now, $\quad {}^{21}C_r = {}^{21}C_{r+1} \Rightarrow {}^{21}C_{21-r} = {}^{21}C_{r+1} \qquad [\because\ {}^{21}C_r = {}^{21}C_{21-r}]$

$\Rightarrow \quad 21 - r = r + 1 \Rightarrow 2r = 20 \Rightarrow r = 10.$

S9. Suppose x^5 occurs in the $(r + 1)^{th}$ term of the expansion $(x + 3)^8$

Now,
$$T_{r+1} = {}^nC_r a^{n-r} b^r$$
$$= {}^nC_r x^{8-r} 3^r$$

Comparing the indices of x in x^5 and in T_{r+1}, we get $r = 3$

Thus, the coefficient of x^5 is

$$^8C_3 (3)^3 = 1512.$$

S10. Suppose x^6y^3 occure in the $(r + 1)^{th}$ term of the expansion $(x + 2y)^9$.

Now,
$$T_{r+1} = {}^9C_r x^{n-r} (2y)^r = {}^9C_r 2^r . x^{9-r} . y^r$$

Comparing the indices of x as well as y in x^6y^3 and in T_{r+1}, we get $r = 3$.

Thus, the coefficient of x^6y^3 is

$${}^9C_3 2^3 = \frac{9!}{3!6!} \cdot 2^3 = \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} \cdot 2^3 = 672 .$$

S11. The coefficients of $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms of the expansion $(1 + x)^{34}$ are ${}^{34}C_{r-6}$ and ${}^{34}C_{2r-2}$, respectively. Since, they are equal so ${}^{34}C_{r-6} = {}^{34}C_{2r-2}$

Therefore, either $r - 6 = 2r - 2$ or $r - 6 = 34 - (2r - 2)$

[Using the fact that if ${}^nC_r = {}^nC_p$, then either $r = p$ or $r = n - p$]

So, we get $r = -4$ or $r = 14$. r being a natural number, $r = -4$ is not possible. So, $r = 14$.

S12.
$$T_{r+1} = {}^6C_r (3x)^{6-r} \left(-\frac{1}{x}\right)^r$$
$$= (-1)^r \cdot {}^6C_r \cdot 3^{6-r} \cdot x^{6-2r}$$

Now, $\quad 6 - 2r = 2 \Rightarrow r = 2$

$\Rightarrow \quad T_3 = (-1)^2 \cdot {}^6C_2 \cdot 3^4 x^2 = 1215\, x^2$

$\Rightarrow$ The required coefficient is 1215.

S13.
$$T_{r+1} = {}^{11}C_r \cdot 5^{11-r} \cdot (-2y)^r$$

Now, $r = 9$

$\Rightarrow \quad T_{10} = (-2)^9 . {}^{11}C_9 \cdot 5^2 \cdot y^9$

$$= -2^9 \cdot 55 \cdot 25 \cdot y^9 = -704000y^9$$

$\Rightarrow$ The required coefficient is – 704000.

S14. We have,

$$(1 + x + x^2 + x^3)^6 = \{(1 + x) + x^2(1 + x)\}^6 = \{(1 + x)(1 + x^2)\}^6 = (1 + x)^6(1 + x^2)^6$$

$$= \{{}^6C_0 + {}^6C_1 x + {}^6C_2 x^2 + \ldots + {}^6C_6 x^6\} \times \{{}^6C_0 + {}^6C_1 x^2 + {}^6C_2 x^4 + \ldots + {}^6C_6 x^{12}\}$$

$\therefore$ Coefficient of x^{14} in this expansion

$$= ({}^6C_2 \times {}^6C_6) + ({}^6C_4 \times {}^6C_5) + ({}^6C_6 \times {}^6C_4)$$

$$= (15 \times 1) + (15 \times 6) + (1 \times 15) = 120.$$

S15. We have,

$$(1 - x - x^2 + x^3)^6 = \{(1 - x) - x^2(1 - x)\}^6 = \{(1 - x)(1 - x^2)\}^6 = (1 - x)^6(1 - x^2)^6$$

$$= \{1 - {}^6C_1 x + {}^6C_2 x^2 - \ldots + {}^6C_6 x^6\}\{1 - {}^6C_1 x^2 + {}^6C_2 x^4 - \ldots + {}^6C_6 x^{12}\}$$

$\therefore$ Coefficient of x^7 in this expansion

$$= (-{}^6C_1 \times -{}^6C_3) + (-{}^6C_3 \times -{}^6C_2) + (-{}^6C_5 \times -{}^6C_1)$$

$$= \{(-6) \times (-20)\} + \{(-20) \times 15\} + \{(-6) \times (-6)\} = -144.$$

S16.

$$\left(\frac{1-x}{1+x}\right)^2 = (1 - x)^2(1 - x)^{-2}$$

$$= (1 - 2x + x^2)(1 - 2x + 3x^2 - 4x^3 + 5x^4 - \ldots)$$

Clearly, the coefficient of x^4 in this expansion

$$= (1 \times 5) + \{(-2) \times (-4)\} + (1 \times 3) = 5 + 8 + 3 = 16.$$

S17. By Binomial Expansion:

$$(1 + x)^n\left(1+\frac{1}{x}\right)^n = (1 + C_1 x + C_2 x^2 + \ldots + C_n x^n) \cdot \left\{1+C_1\left(\frac{1}{x}\right)+C_2\left(\frac{1}{x^2}\right)+\ldots+C_n\left(\frac{1}{x^n}\right)\right\}$$

$$= (1 + C_1 x + C_2 x^2 + \ldots + C_n x^n) \cdot (1 + C_1 x^{-1} + C_2 x^{-2} + \ldots + C_2 x^{-n})$$

$\therefore$ The coefficient of x^{-n} in $(1 + x)^n\left(1+\frac{1}{x}\right)^n$ is C_n *i.e.*, 1. $\quad [\because\ C_n = {}^nC_n = 1]$

S18. By Binomial Expansion:

$$(1 + x)(1 - x)^n = (1 + x) \times \{1 - {}^nC_1 x + {}^nC_2 x^2 - \ldots + (-1)^{n-1}\,{}^nC_{n-1} x^{n-1} + (-1)^n\,{}^nC_n x^n\}$$

Clearly, the coefficient of x^n in this expansion

$$= (-1)^n\,{}^nC_n + (-1)^{n-1}\,{}^nC_{n-1}$$

$$= (-1)^n + (-1)^{n-1}.n \qquad [\because\ {}^nC_n = 1,\ {}^nC_{n-1} = n]$$

$$= (-1)^n(1 - n).$$

S19. We have,

$$(1 + x^2)^5(1 + x)^4 = \{^5C_0 + {}^5C_1 (x^2) + {}^5C_2 (x^2)^2 + {}^5C_3 (x^2)^3 + {}^5C_4 (x^2)^4 + {}^5C_5 (x^2)^5\}$$
$$\times \{^4C_0 + {}^4C_1\, x + {}^4C_2\, x^2 + {}^4C_3\, x^3 + {}^4C_4\, x^4\}$$

$\therefore$ The coefficient of x^5 in this expansion

$$= (^5C_1 \times {}^4C_3) + (^5C_2 \times {}^4C_3)$$
$$= (5 \times 4) + (10 \times 4) = 60. \qquad [\because\ {}^5C_1 = 5,\ {}^5C_2 = 10,\ {}^4C_3 = {}^4C_1 = 4]$$

S20. By Binomial Expansion:

$(1 + x^2)^{12}\,(1 + x^{12})\,(1 + x^{24})$

$$= (^{12}C_0 + {}^{12}C_1\, x^2 + {}^{12}C_2\, x^4 + \ldots + {}^{12}C_6\, x^{12} + \ldots + {}^{12}C_{12}\, x^{24})\ .\ (1 + x^{12} + x^{24} + x^{36})$$

$\therefore$ The coefficient of x^{24} in this expansion

$$= {}^{12}C_0 + {}^{12}C_6 + {}^{12}C_{12} = {}^{12}C_6 + 2 = 926 \qquad [\because\ {}^{12}C_0 = {}^{12}C_{12} = 1]$$

S21. The general term in the expansion of $\left(x^2 + \dfrac{a}{x}\right)^5$ is

$$t_{r+1} = {}^5C_r\, (x^2)^{5-r} \left(\frac{a}{x}\right)^r = {}^5C_r\, a^r\, x^{10-3r} \qquad \ldots \text{(i)}$$

This term will contain x if $10 - 3r = 1$, *i.e.*, if $r = 3$.

Putting $r = 3$ in Eq. (i), we get

$$t_4 = {}^5C_3\, a^3\, x^1 = {}^5C_3\, a^3\, x$$

The coefficient of $x = {}^5C_3\, a^3 = 10a^3$.

S22. The r^{th} term in the expansion of $\left(\dfrac{x}{3} - \dfrac{2}{x^2}\right)^{10}$ is

$$t_r = (-1)^{r-1}\ {}^{10}C_{r-1} \left(\frac{x}{3}\right)^{10-(r-1)} \left(\frac{2}{x^2}\right)^{r-1}$$
$$= (-1)^{r-1}.{}^{10}C_{r-1}(3)^{r-11}\, 2^{r-1}\ .\ x^{13-3r}$$

Now, this term contains x^4.

$\therefore \qquad 13 - 3r = 4 \quad \Rightarrow \quad 9 = 3r \quad \Rightarrow \quad r = 3.$

S23. Given that coefficient of T_4 = coefficient of T_9

$$T_{r+1} = {}^nC_r\, (x^2)^{n-r} \left(\frac{1}{x}\right)^r$$

and
$$T_{r+1} = {}^nC_r\, x^{2n-3r}$$
$$T_4 = {}^nC_3\, x^{2n-9} \text{ and } T_9 = {}^nC_8\, x^{2n-24}$$

We know that if ${}^nC_p = {}^nC_q$ then $n = p + q$ (Here $p \neq q$)

$$n = 3 + 8 = 11,\ T_6 = {}^{11}C_5 x^{22-15} = 462\ x^7$$

S24.

$$T_{r+1} = (-1)^r\ {}^{15}C_r\ (x^4)^{15-r}\left(\frac{1}{x^3}\right)^r$$

$$= (-1)^r\ {}^{15}C_r\, x^{60-4r-3r} = (-1)^r\ {}^{15}C_r\, x^{60-7r}$$

Here $60 - 7r = -17 \Rightarrow 77 = 7r \Rightarrow r = 11,\qquad T_{12} = (-1)^{11}\ {}^{15}C_{11} x^{60-77}$

Coefficient of $\frac{1}{x^{17}} = \frac{-15\times 14\times 13\times 12\times 11!}{11!\times 4\times 3\times 2} = -1365$

S25. We have,

$$(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1\, x + {}^{20}C_2\, x^2 + \dots + {}^{20}C_{20}\, x^{20}$$

$\therefore$ The coefficient of r^{th} term = ${}^{20}C_{r-1}$ and coefficient of $(r + 1)^{th}$ term = ${}^{20}C_r$.

Now, $\frac{{}^{20}C_{r-1}}{{}^{20}C_r} = \frac{1}{2} \Rightarrow \frac{20!}{(r-1)!(21-r)!}\times\frac{r!(20-r)!}{20!} = \frac{1}{2}$

$\Rightarrow \frac{r}{21-r} = \frac{1}{2} \Rightarrow 2r = 21 - r \Rightarrow r = 7.$

S26. Let the term containing x^{32} be $(r + 1)^{th}$ term we have.

$$t_{r+1} = (-1)^r.\ {}^{15}C_r\,(x^4)^{15-r}\left(\frac{1}{x^3}\right)^r$$

$$= (-1)^r.{}^{15}C_r\,.\ x^{(60-7r)}$$

Put $60 - 7r = 32$ we get $r = 4$.

$\therefore$ $(4 + 1)^{th} \Rightarrow 5^{th}$ term contains x^{32}.

$\therefore$ $t_5 = (-1)^4\,.\ {}^{15}C_4\,.\ x^{32}$

$\therefore$ $t_5 = (-1)^4\,.\ {}^{15}C_4\, x^{32}$

$$= \left(\frac{15\times 14\times 13\times 12}{4\times 3\times 2\times 1\times}\right)x^{32} = 1365\ x^{32}.$$

$\therefore$ Coefficient of x^{32} in the given expression is 1365.

S27. General term in the expansion of $(x + 3)^8$ is $T_{r+1} = {}^8C_r x^{8-r}(3)^r$

Putting $8 - r = 5$ *i.e.* $r = 3$ in above, we get

$$T_{3+1} = T_4 = {}^8C_3 x^{8-3}(3)^3$$

$$= {}^8C_3\, x^5 \,.\, 27$$

Hence, the required coefficient is

$$= \frac{8!}{3!(8-3)!}\cdot(27) = \frac{8!}{3!\times 5!}\times(27)$$

$$= \frac{8\times 7\times 6\times 5!}{1\times 2\times 3\times 5!}(27) = 8\times 7\times 27 = 1512$$

S28. The general term in the expansion of $[a + (-2b)]^{12}$ is

$$T_{r+1} = {}^{12}c_r\, a^{12-r}(-2b)^r \qquad \text{...(i)}$$

Putting $12 - r = 5$ i.e., $r = 7$ in (i), we get

$$T_{7+1} = {}^{12}C_7 a^{12-7}(-2b)^7 = {}^{12}C_7 a^5(-2)^7 b^7 = {}^{12}C_7(-2)^7 a^5 b^7$$

Hence, the required coefficient is

$${}^{12}c_7\,(-2)^7 = \frac{-12!}{7!(12-7)!}\cdot 2^7 \qquad = -101376$$

S29. We have, $(1 + 2x)^6\,(1 - x)^7 = [1 + {}^6C_1\,(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6]$

$$\times\,[1 - {}^7C_1x + {}^7C_2x^2 - {}^7C_3x^3 + {}^7C_4x^4 - {}^7C_5x^5 + \ldots\ldots]$$

$$= [1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + \ldots.]$$

$$\times\,[1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + \ldots.]$$

Therefore, coefficient of x^5 in the product

$$= 1\times(-21) + 12\times 35 + 60\,(-35) + 160\times 21 + 240\times(-7) + 192\times 1$$

$$= -21 + 420 - 2100 + 3360 - 1680 + 192 = 171$$

S30. Let C_1 and C_2 be the coefficients of x^n in the binomial expansions of $(1 + x)^{2n}$ and $(1 + x)^{2n-1}$ respectively. Then,

$$C_1 = \text{Coefficient of } x^n \text{ in } (1+x)^{2n} = {}^{2n}C_n = \frac{(2n)!}{n!\,n!} = \frac{(2n)(2n-1)!}{n(n-1)!\,n!}$$

$$\Rightarrow \qquad C_1 = 2\frac{(2n-1)!}{(n-1)!\,n!} \qquad \text{... (i)}$$

and, $\quad C_2 = \text{coefficient of } x^n \text{ in } (1 + x)^{2n-1}$

$$\Rightarrow \qquad C_2 = {}^{2n-1}C_n = \frac{(2n-1)!}{(n-1)!\,n!} \qquad \text{... (ii)}$$

From (i) and (ii), we get

$$C_1 = 2C_2$$

Therefore, coefficient of x^n in $(1 + x)^{2n} = 2\times$ coefficient of x^n in $(1 + x)^{2n-1}$.

S31. General term in the expansion of $(a + b)^n$ is

$$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r \quad \dots \text{(i)}$$

Putting $r + 1 = 4$ *i.e.* $r = 3$ in (i), we get

$$T_4 = T_{3+1} = {}^nC_3 a^{n-3}\, b^3 \quad \dots \text{(ii)}$$

Again putting $r + 1 = 13$ i.e., $r = 12$ in (i), we get

$$T_{13} = T_{12+1} = {}^nC_{12}\, a^{n-12}\, b^{12} \quad \dots \text{(iii)}$$

According to the question,

Coefficient of T_4 = coefficient of T_{13}

$\Rightarrow \quad {}^nC_3 = {}^nC_{12}$ [Using (ii) and (iii)]

$\Rightarrow \quad {}^nC_3 = {}^nC_{n-12}$ $[\because {}^nC_r = {}^nC_{n-r}]$

$\Rightarrow \quad 3 = n - 12$

$\Rightarrow \quad n = 15.$

S32. Here $n = 8$(even)

$$\text{Middle term} = \left(\frac{n}{2} + 1\right)\text{th} = 5^{\text{th}} \text{ term}, \; T_5 = {}^8C_4(1)^4\, a^4 = {}^8C_4 a^4$$

Middle term in $(1 + a)^7$, $n = 7$(odd)

4^{th} and 5^{th} terms $T_4 = {}^7C_3 a^3,\; T_5 = {}^7C_4\, a^4$

We have to prove ${}^8C_4 = {}^7C_3 + {}^7C_4$

$$\text{L.H.S.} = \frac{8!}{4!\,4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2} = 70$$

$$\text{R.H.S.} = {}^7C_3 + {}^7C_4 = \frac{7!}{3!\,4!} + \frac{7!}{3!\,4!} = \frac{2.7!}{3!\,4!} = 70,$$

L.H.S. = R.H.S.

S33. $T_2 = {}^{2n}C_1 x,\; T_3 = {}^{2n}C_2 x^2$ and $T_4 = {}^{2n}C_3 x^3$

Coefficients are in A.P.

$\therefore \quad T_4 - T_3 = T_3 - T_2,$

$${}^{2n}C_3 - {}^{2n}C_2 = {}^{2n}C_2 - {}^{2n}C_1$$

$$2\,({}^{2n}C_2) = {}^{2n}C_1 + {}^{2n}C_3$$

$$\Rightarrow \quad 2\,\frac{2n!}{2!(2n-2)!} = \frac{2n!}{2!(2n-1)!} + \frac{2n!}{3!(2n-3)!}$$

$$\Rightarrow \quad 2n\,(2n - 1) = (2n) + \frac{(2n)(2n-1)(2n-2)}{6}$$

$\Rightarrow \quad 6(2n-1) = 6 + (2n-1)(2n+2) \quad \Rightarrow \quad 12n - 6 = 6 + 4n^2 - 6n + 2$

$\Rightarrow \quad 4n^2 - 18n + 14 = 0 \quad \Rightarrow \quad 2n^2 - 9n + 7 = 0$

$\Rightarrow \quad 2n^2 - 2n + 7n + 7 = 0 \quad \Rightarrow \quad 2n(n-1) - 7(n-1) = 0$

$\Rightarrow \quad (n-1)(2n-7) = 0 \quad \Rightarrow \quad n = 1$ or $2n = 7$, $\quad$ ($n = 1$ is not possible)

$\Rightarrow \quad n = \frac{7}{2}$

S34.

$$T_r = {}^nC_{r-1}x^{r-1},\ T_{r+1} = {}^nC_r x^r,\ T_{r+2} = {}^nC_{r+1}\ x^{r+1}$$

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6:33:110,\ \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{33}{6}$$

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{11}{2} \quad \Rightarrow \quad \frac{(r-1)!(n-r+1)(n-r)!}{r(r-1)!(n-r)!} = \frac{11}{2}$$

$$\frac{n-r+1}{r} = \frac{11}{2} \quad \Rightarrow \quad 2n - 2r + 2 = 11r \quad \Rightarrow \quad 2n + 2 = 13r \qquad \text{... (i)}$$

$$\frac{r!(n-r)(n-r-1)!}{(r+1)r!(n-r-1)!} = \frac{10}{3}, \qquad \frac{n-r}{r+1} = \frac{10}{3}$$

$$3n - 3r = 10r + 10 \quad \Rightarrow \quad 3n - 13r = 10 \qquad \text{... (ii)}$$

After solving Eq. (i) and (ii), we get

$$n = 12.$$

S35.

The general term in the expansion of $\left(\frac{x^2}{2} - \frac{2}{x}\right)^9$ is

$$t_{r+1} = {}^9C_r\ (-1)^r \left(\frac{x^2}{2}\right)^{9-r} \left(\frac{2}{x}\right)^r$$

$$= {}^9C_r (-1)^r\ (2)^{2r-9}\ x^{18-3r} \qquad \text{... (i)}$$

This term will contain x^{-9} if $18 - 3r = -9$, *i.e.*, if $r = 9$.

Putting $r = 9$ in Eq. (i), we get

$$t_{10} = {}^9C_9 (-1)^9\ (2)^9\ x^{-9} = -2^9\ x^{-9} = -512\, x^{-9}.$$

$\therefore$ The coefficient of x^{-9} in this expansion is -512.

S36. $(1 + x + x^3 + x^4)^{10} = \{1 + x + x^3(1 + x)\}^{10} = \{(1 + x)(1 + x^3)\}^{10} = (1 + x)^{10}(1 + x^3)^{10}$

$$= \{{}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + {}^{10}C_3 x^3 + {}^{10}C_4 x^4 + \dots + {}^{10}C_{10} x^{10}\}$$

$$\times \{{}^{10}C_0 + {}^{10}C_1 x^3 + {}^{10}C_2 x^6 + \dots + {}^{10}C_{10} x^{30}\}$$

The coefficient of x^4 in this expansion

$$= ({}^{10}C_1 \times {}^{10}C_1) + ({}^{10}C_4 \times {}^{10}C_0)$$

$$= (10 \times 10) + \left(\frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times 1\right) = 310.$$

S37. The general term in the expansion of $\left(\frac{1}{3}x^{1/2} + x^{-1/4}\right)^{10}$ is

$$t_{r+1} = {}^{10}C_r \left(\frac{1}{3}x^{1/2}\right)^{10-r} (x^{-1/4})^r = {}^{10}C_r\,(3)^{r-10}\,x^{5-\frac{3}{4}r} \quad \text{... (i)}$$

Now, this term contains x^2 if $5 - \frac{3}{4}r = 2$, *i.e.*, if $r = 4$.

Putting $r = 4$ in Eq. (i), we get

$$t_5 = {}^{10}C_4\,(3)^{-6}\,x^2 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times \frac{1}{3^6}\,x^2 = \frac{70}{243}x^2$$

$\therefore$ The coefficient of x^2 term in this expansion is $\frac{70}{243}$.

S38. The general term in the expansion of $\left(\sqrt{x} - \frac{2}{x}\right)^{17}$ is

$$t_{r+1} = (-1)^r\ {}^{17}C_r\,(\sqrt{x})^{17-r}\left(\frac{2}{x}\right)^r = (-1)^r\ {}^{17}C_r \cdot 2^r \cdot x^{\left(\frac{17-3r}{2}\right)} \quad \text{... (i)}$$

Putting $\frac{17-3r}{2} = -11$, we get,

$$r = 13.$$

Putting $r = 13$ in Eq. (i), we get,

$$t_{14} = (-1)^{13}\ {}^{17}C_{13} \cdot 2^{13}\,x^{-11} = {}^{17}C_{13}(-2)^{13} \cdot x^{-11}$$

$\therefore$ The coefficient of x^{-11} in the expansion of $\left(\sqrt{x} - \frac{2}{x}\right)^{17}$ is ${}^{17}C_{13}(-2)^{13}$.

S39. We have,

$$(1 + 3x + 2x^2)^6 = \{(1 + x)(1 + 2x)\}^6 = \{(1 + x)^6 (1 + 2x)^6$$

$$= \{{}^6C_0 + {}^6C_1 x + {}^6C_2 x^2 + \ldots + {}^6C_6 x^6\}$$

$$\times \{{}^6C_0 + {}^6C_1 (2x) + {}^6C_2 (2x)^2 + \ldots + {}^6C_6 (2x)^6\}$$

$$= \{{}^6C_0 + {}^6C_1 x + {}^6C_2 x^2 + \ldots + {}^6C_5 x^5 + {}^6C_6 x^6\}$$

$$\times \{{}^6C_0 + {}^6C_1 (2)(x) + {}^6C_2 (2)^2(x)^2 + \ldots + {}^6C_5 (2)^5(x)^5 + {}^6C_6 (2)^6(x)^6\}$$

$\therefore$ Coefficient of x^{11} in this expansion

$$= \{^6C_5 \times {}^6C_6(2)^6\} + \{^6C_6 \times {}^6C_5(2)^5\} = 576.$$

S40. We have,

$$(1 + 2x + 3x^2 + \dots)^{3/2} = \{(1 - x)^{-2}\}^{3/2} \qquad [\because\ (1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots]$$

$$= (1 - x)^{-3}$$

$$= 1 + 3x + \frac{3.4}{2!}x^2 + \frac{3.4.5}{3!}x^3 + \frac{3.4.5.6}{4!}x^4 + \frac{3.4.5.6.7}{5!}x^5 + \dots$$

$$\left[\because\ (1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \dots\right]$$

$\therefore$ Coefficient of $x^5 = \dfrac{3.4.5.6.7}{5!} = 21$.

S41. $(1 + x + x^2)^n = \{1 + (x + x^2)\}^n$

$$= {}^nC_0 + {}^nC_1(x + x^2) + {}^nC_2(x + x^2)^2 + {}^nC_3(x + x^2)^3 + \dots$$

$$= 1 + {}^nC_1\, x(1 + x) + {}^nC_2\, x^2(1 + x)^2 + {}^nC_3\, x^3(1 + x)^3 + \dots$$

$$= 1 + nx(1+x) + \frac{n(n-1)}{2!}\cdot x^2(1 + 2x + x^2) + \frac{n(n-1)(n-2)}{3!}x^3(1 + 3x + 3x^2 + x^3) + \dots$$

$\therefore$ Coefficient of x^3 in this expansion

$$= \frac{n(n-1)}{2!}\cdot 2 + \frac{n(n-1)(n-2)}{3!}\cdot 1$$

$$= n(n-1) + \frac{n(n-1)(n-2)}{6}$$

$$= \frac{n(n-1)(n+4)}{6}.$$

S42. We have

$$(1 + x^2 - x^3)^8 = \{1 + x^2(1 - x)\}^8$$

$$= 1 + {}^8C_1\, x^2(1 - x) + {}^8C_2\, x^4(1 - x)^2 + {}^8C_3\, x^6(1 - x)^3 + {}^8C_4\, x^8(1 - x)^4 + {}^8C_5\, x^{10}(1 - x)^5 + \dots + {}^8C_8\, x^{16}(1 - x)^8$$

Out of these terms, the terms containing x^{10} are ${}^8C_4\, x^8(1 - x)^4$ and ${}^8C_5\, x^{10}(1 - x)^5$

i.e., ${}^8C_4\, x^8(1 - 4x + 6x^2 - 4x^3 + x^4)$ and ${}^8C_5\, x^{10}(1 - 5x + \dots)$

$$\left[\begin{aligned} \because\quad (1-x)^4 &= 1 - {}^4C_1 x + {}^4C_2 x^2 - {}^4C_3 x^3 + {}^4C_4 x^4 \\ &= 1 - 4x + 6x^2 - 4x^3 + x^4 \\ \text{and}\quad (1-x)^5 &= 1 - {}^5C_1 x + {}^5C_2 x^2 - \dots + {}^5C_5 x^5 \\ &= 1 - 5x + 10x^2 - \dots + x^5 \end{aligned}\right]$$

Clearly, the coefficient of x^{10} in the given expansion

$$= {}^8C_4 \cdot (6) + {}^8C_5$$

$$= \left(\frac{8\times7\times6\times5}{4\times3\times2\times1}\times 6\right)+\left(\frac{8\times7\times6}{3\times2}\right) = 476. \qquad [\because\ {}^8C_5 = {}^8C_3]$$

S43. We have,

$$P = 1 + (1 + x) + (1 + x)^2 + ... + (1 + x)^n$$

$$= \frac{(1+x)^{n+1}-1}{(1+x)-1} \qquad \left[\begin{array}{l}\text{Sum to } (n+1) \text{ terms of a G.P. with}\\ \text{first term} = 1 \text{ and common ratio} = (1+x)\end{array}\right]$$

$$= \frac{1}{x}\{(1+x)^{n+1}-1\}$$

$$= \frac{1}{x}\ [\{1 + {}^{n+1}C_1\,x + {}^{n+1}C_2\,x^2 + \dots + {}^{n+1}C_r\,x^r + {}^{n+1}C_{r+1}\,x^{r+1} + \dots + x^{n+1}\} - 1]$$

$$= \{{}^{n+1}C_1 + {}^{n+1}C_2\,x + \dots + {}^{n+1}C_r\,x^{r-1} + {}^{n+1}C_{r+1}\,x^r + \dots + x^n\}$$

Thus, the coefficient of x^r in this expansion is ${}^{n+1}C_{r+1}$ or ${}^{n+1}C_{n-r}$.

S44. The general term in the expansion of $(3 + ax)^9$ is

$$t_{r+1} = {}^9C_r\,(3)^{9-r}\,(ax)^r = {}^9C_r\,(3)^{9-r}\,a^r\,x^r \qquad \dots \text{(i)}$$

Putting $r = 2$ in Eq. (i), we get

$$t_3 = {}^9C_2\,(3)^7\,a^2\,x^2$$

Putting $r = 3$ in Eq. (i), we get

$$t_4 = {}^9C_3\,(3)^6\,a^3\,x^3$$

$\therefore$ The coefficients of x^2 and x^3 are ${}^9C_2\,3^7\,a^2$ and ${}^9C_3\,3^6\,a^3$ respectively.

Now,

$${}^9C_2\,3^7\,a^2 = {}^9C_3\,3^6\,a^3$$

$$\Rightarrow \qquad \frac{9\times8}{2}\times3^7\,a^2 = \frac{9\times8\times7}{3\times2}\times3^6\,a^3$$

$$\Rightarrow \qquad a = \frac{9\times8}{2}\times3^7\times\frac{3\times2}{9\times8\times7\times3^6} = \frac{9}{7}.$$

S45. The general term in the expansion of $\left(\sqrt{x^5}+\frac{3}{\sqrt{x^3}}\right)^6$ is

$$t_{r+1} = {}^6C_r\,(\sqrt{x^5})^r\left(\frac{3}{\sqrt{x^3}}\right)^{6-r} = {}^6C_r\,(3)^{6-r}\,x^{4r-9} \qquad \dots \text{(i)}$$

If this term contains x^3, then we have: $4r - 9 = 3 \Rightarrow r = 3$.

Putting $r = 3$ in Eq. (i), we get

$$t_4 = {}^6C_3\,(3)^3\,x^3 = \frac{6 \times 5 \times 4}{3 \times 2} \times 27 \times x^3 = 540\,x^3.$$

$\therefore$ The coefficient of x^3 is 540.

S46. The general term in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is

$$t_{r+1} = {}^{10}C_r(-1)^r \left(\frac{x}{2}\right)^{10-r} \left(\frac{3}{x^2}\right)^r = {}^{10}C_r(-1)^r\, 2^{r-10}\, 3^r \cdot x^{10-3r} \quad \text{... (i)}$$

This term contains x^4 if $10 - 3r = 4$, *i.e.*, if $r = 2$.

$\therefore$ The term containing x^4 is

$$t_3 = {}^{10}C_2\,(-1)^2\,2^{(-8)}\,3^2\,x^4 \quad \text{[Putting } r = 2 \text{ in Eq. (i)]}$$

$$t_3 = \frac{10 \times 9}{2} \times 1 \times \frac{9}{256} \times x^4 = \frac{405}{256}\,x^4.$$

Thus, the coefficient of x^4 in the expansion of $\left(\frac{x}{2} - \frac{3}{x^2}\right)^{10}$ is $\frac{405}{256}$.

S47. The general term in the expansion of $(1 + a + b - c)^{10}$ is

$$T = \frac{10!}{p!q!r!s!} \cdot (1)^p (a)^q (b)^r (-c)^s \quad \text{where} \quad p + q + r + s = 10$$

$$T = \frac{10!}{p!q!r!s!} \cdot (-1)^s (a)^q (b)^r (c)^s \quad \text{where} \quad p + q + r + s = 10 \quad \text{... (i)}$$

If this term contains $a^3b^4c^2$, then $q = 3$, $r = 4$, $s = 2$

Putting these values of p, q, r and s in Eq. (i), we get the term containing $a^3b^4c^2$ as

$$T = \frac{10!}{1!3!4!2!}(-1)^2 (a)^3 (b)^4 (c)^2$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5}{3 \times 2}\,a^3b^4c^2$$

$\Rightarrow$ $T = 12600\,a^3b^4c^2$.

Hence, the coefficient of $a^3b^4c^2$ is 12600.

S48. We have, $(1 + x)^6 + (1 + x)^7 + \ldots + (1 + x)^{15}$

$$= \frac{(1+x)^6\ \{(1+x)^{10} - 1\}}{(1+x) - 1}$$

$$= \frac{1}{x}\{(1+x)^{16} - (1+x)^6\}$$

$$= \frac{1}{x}\ \{({}^{16}C_0 + {}^{16}C_1 x + {}^{16}C_2 x^2 + \dots + {}^{16}C_7 x^7 + \dots + {}^{16}C_{16} x^{16})$$
$$- ({}^{6}C_0 + {}^{6}C_1 x + {}^{6}C_2 x^2 + \dots + {}^{6}C_6 x^6)\}$$

$$= ({}^{16}C_0 - {}^{6}C_0)\ \frac{1}{x} + ({}^{16}C_1 - {}^{6}C_1) + ({}^{16}C_2 - {}^{6}C_2)\, x + \dots + ({}^{16}C_6 - {}^{6}C_6)\, x^5$$
$$+ {}^{16}C_7 x^6 + \dots + {}^{16}C_{16} x^{15} \qquad [\because\ {}^{n}C_r = {}^{n}C_{n-r}]$$

$\therefore$ The coefficient of x^6 in the expansion of

$$(1 + x)^6 + (1 + x)^7 + \dots + (1 + x)^{15} \text{ is } {}^{16}C_7 = {}^{16}C_9 = 11440$$

S49. We have,

$$(1 + x)^n = {}^{n}C_0 + {}^{n}C_1 x + {}^{n}C_2 x^2 + \dots + {}^{n}C_n x^n.$$

The general term of this expansion is

$$t_{r+1} = {}^{n}C_r X_r$$

$\therefore$ Coefficient of $t_p = {}^{n}C_{p-1}$ and the coefficient of $t_{p+1} = {}^{n}C_p$.

So, we have

$${}^{n}C_{p-1} = p \quad \text{and} \quad {}^{n}C_p = q$$

$$\therefore \qquad \frac{q}{p} = \frac{{}^{n}C_p}{{}^{n}C_{p-1}} = \frac{n-p+1}{p} \qquad \left[\because\ \frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \qquad \frac{q}{p} + 1 = \frac{n-p+1}{p} + 1 \quad \Rightarrow \quad \frac{q+p}{p} = \frac{n+1}{p} \quad \Rightarrow \quad q + p = n + 1.$$

$$\Rightarrow \qquad p + q = n + 1$$

S50. We have, $(1 + x)^{21} + (1 + x)^{22} + \dots + (1 + x)^{30}$

$$= \frac{(1+x)^{21}\{(1+x)^{10} - 1\}}{(1+x) - 1}$$

$$= \frac{1}{x}\{(1+x)^{31} - (1+x)^{21}\}$$

Now, the coefficient of x^5 in the given expansion

$$= \text{Coeff. of } x^5 \text{ in } \frac{1}{x}\{(1 + x)^{31} - (1 + x)^{21}\}$$

$$= \text{Coeff. of } x^6 \text{ in } \{(1 + x)^{31} - (1 + x)^{21}\} = {}^{31}C_6 - {}^{21}C_6 = 682017$$

$$[\because\ \text{Coeff. of } x^6 \text{ in } (1 + x)^{31} \text{ is } {}^{31}C_6 \text{ and in } (1 + x)^{21} \text{ is } {}^{21}C_6]$$

S51. By Binomial Expansion:

$$(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2 + \dots + {}^{20}C_{20} x^{20}$$

The general term in this expansion is given by

$t_{r+1} = {}^{20}C_r x^r$. Its coefficient is ${}^{20}C_r$. $[\because\ {}^nC_m = {}^nC_{n-m}]$

$\therefore$ Coefficient of $(m + 1)^{th}$ term = coefficient of $(m + 3)^{th}$ term.

$$\Rightarrow \quad {}^{20}C_m = {}^{20}C_{m+2} \quad \Rightarrow \quad {}^{20}C_{20-m} = {}^{20}C_{m+2} \qquad [\because\ {}^{20}C_m = {}^{20}C_{20-m}]$$

$$\Rightarrow \quad 20 - m = m + 2 \quad \Rightarrow \quad 2m = 20 - 2 \quad \Rightarrow \quad m = 9.$$

S52. The middle term in the expansion of $(1 + x)^{2n}$ will be T_{n+1} and its coefficient will be

$$ {}^{2n}C_n = \frac{(2n)!}{(n!)^2} \qquad \text{... (i)}$$

The middle term in the expansion of $(1 + x)^{2n-1}$ will be T_n and T_{n+1}. The sum of their coefficient will be

$$ {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = \frac{(2n-1)!}{(n-1)!\cdot(n!)} + \frac{(2n-1)!}{n!\cdot(n-1)!} = 2\cdot\frac{(2n-1)!}{(n-1)!\cdot(n!)}$$

$$ = \frac{(2n)\cdot(2n-1)!}{n(n-1)!(n!)} = \frac{(2n)!}{(n!)^2} \qquad \text{... (ii)}$$

From Eq. (i) (ii), we get the result.

S53. Let, $bc = x, \quad ca = y, \quad ab = z$

Then, $(bc + ca + ab)^9 = (x + y + z)^9$

The general term in the expansion of $(x + y + z)^9$ is

$$T = \frac{9!}{p!q!r!}\cdot x^p y^q x^r \qquad \text{where} \quad p + q + r = 9$$

$$\Rightarrow \quad T = \frac{9!}{p!q!r!}(bc)^p(ca)^q(ab)^r \qquad \text{where} \quad p + q + r = 9$$

$$\Rightarrow \quad T = \frac{9!}{p!q!r!}\, a^{q+r} b^{r+p} c^{p+q} \qquad \text{where} \quad p + q + r = 9 \qquad \text{... (i)}$$

If this term contains $a^5b^6c^7$, then

$$q + r = 5, \quad r + p = 6, \quad p + q = 7, \quad p + q + r = 9$$

i.e., $p = 4, \quad q = 3, \quad r = 2$

Putting these values of p, q, r in Eq. (i), we get the term containing $a^5b^6c^7$ as

$$T = \frac{9!}{4!3!2!}\, a^5b^6c^7 = 1260\, a^5b^6c^7.$$

Hence, the coefficient of $a^5b^6c^7$ in the expansion of $(bc + ca + ab)^9$ is 1260.

S54. The general term in the expansion of $(a - b + c - d)^{10}$ is

$$T = \frac{10!}{p!q!r!s!}\cdot a^p\cdot(-b)^q\cdot c^r\cdot(-d)^s \qquad \text{where} \quad p + q + r + s = 10 \qquad \text{... (i)}$$

If this term contains $a^4b^3c^2d$, then $p = 4$, $q = 3$, $r = 2$, $s = 1$

Putting these values of p, q, r and s in Eq. (i), we get the term containing $a^4b^3c^2d$ as

$$T = \frac{10!}{4!3!2!1!} \cdot a^4 \cdot (-b)^3 \cdot c^2 \cdot (-d)^1$$

$$\Rightarrow \qquad T = (-1)^4 \times \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(4 \times 3 \times 2 \times 1) \times (3 \times 2 \times 1) \times (2 \times 1) \times 1} a^4b^3c^2d$$

$$\Rightarrow \qquad T = 12600\, a^4b^3c^2d.$$

Hence, the coefficient of $a^4b^3c^2d$ in this expansion is 12600.

S55. We have,

$$(1-4x)^{-1/2} = 1 + \frac{1}{2}(4x) + \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{2!} \cdot (4x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{3!} \cdot (4x)^3 + \ldots + \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\ldots\left(\frac{2r-1}{2}\right)}{r!} \cdot (4x)^r + \ldots$$

The general term in this expansion is given by

$$t_{r+1} = \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\ldots\left(\frac{2r-1}{2}\right)}{r!} \cdot (4x)^r + \ldots$$

$$\Rightarrow \qquad t_{r+1} = \frac{1.3\ldots(2r-1)}{(2^r).(r!)}.2^{2r}\,x^r = \frac{1.3\ldots(2r-1)}{r!}.2^r\,x^r$$

$$\Rightarrow \qquad t_{r+1} = \frac{1.2.3.4\ldots(2r-1).2r}{(2.4.6\ldots 2r)}.\frac{2^r\,x^r}{r!} = \frac{(2r)!}{(r!).2^r}.\frac{2^r.x^r}{(r!)}$$

$$\Rightarrow \qquad t_{r+1} = \frac{(2r)!}{(r!)^2}.x^r \qquad \ldots \text{(i)}$$

If this term contains x^n, then $r = n$.

Putting $r = n$ in Eq. (i), we get

$$t_{n+1} = \frac{(2n)!}{(n!)^2}.x^n$$

Hence, the coefficient of x^n in the expansion is $\frac{(2n)!}{(n!)^2}$.

S56. The coefficients of the first three terms of $\left(x - \frac{3}{x^3}\right)^m$ are ${}^mC_0$. $(-3)\,{}^mC_1$ and $9\,{}^mC_2$. Therefore, by the given condition, we have

$${}^mC_0 - 3\,{}^mC_1 + 9\,{}^mC_2 = 559$$

i.e., $1 - 3m + \frac{9m(m-1)}{2} = 559$

which gives m = 12 (m being a natural number).

Now, $T_{r+1} = {}^{12}C_r\, x^{12} \left(\frac{-3}{x^3}\right)^r = {}^{12}C_r\,(-3)^r\,.\,x^{12-3r}$

Since, we need the term containing x^3, so put $12 - 3r = 3$ *i.e.*, $r = 3$.

Thus, the required term is ${}^{12}C_3\,(-3)^3\,x^3$, *i.e.*, $-5940\,x^3$.

S57. We first expand each of the factors of the given product using Binomial Theorem. We have

$$(1 + 2a)^4 = {}^4C_0 + {}^4C_1\,(2a) + {}^4C_2\,(2a)^2 + {}^4C_3\,(2a)^3 + {}^4C_4\,(2a)^4$$
$$= 1 + 4\,(2a) + 6\,(4a^2) + 4\,(8a^3) + 16a^4$$
$$= 1 + 8a + 24a^2 + 32a^3 + 16a^4$$

and $$(2 - a)^5 = {}^5C_0\,(2)^5 - {}^5C_1\,(2)^4\,(a) + {}^5C_2\,(2)^3\,(a)^2 - {}^5C_3\,(2)^2\,(a)^3 + {}^5C_4\,(2)\,(a)^4 - {}^5C_5\,(a)^5$$
$$= 32 - 80a + 80a^2 - 40a^3 + 10a^4 - a^5$$

Thus, $(1 + 2a)^4\,(2 - a)^5 = (1 + 8a + 24a^2 + 32a^3 + 16a^4)\,(32 - 80a + 80a^2 - 40a^3 + 10a^4 - a^5)$

The complete multiplication of the two brackets need not be carried out. We write only those terms which involve a^4. This can be done if we note that $a^r\,.\,a^{4-r} = a^4$. The terms containing a^4 are

$$1\,(10a^4) + (8a)\,(-40a^3) + (24a^2)\,(80a^2) + (32a^3)\,(-80a) + (16a^4)\,(32) = -438a^4.$$

Thus, the coefficient of a^4 in the given product is -438.

S58. $$T_{r+1} = {}^{2n}C_r\,x^r$$

For coefficient of x put $r = 1$, *i.e.*, ${}^{2n}C_1$

For coefficient of x^2 put $r = 2$, *i.e.*, ${}^{2n}C_2$ For coefficient of x^3 put $r = 3$, *i.e.*, ${}^{2n}C_3$

These are in A.P $2.\,{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3 \Rightarrow \frac{2 \times (2n)!}{2!(2n-2)!} = \frac{(2n)!}{(2n-1)!} + \frac{(2n)!}{6.(2n-3)!}$

$$\Rightarrow \frac{1}{(2n-2)(2n-3)!} = \frac{1}{(2n-1)(2n-2)(2n-3)!} + \frac{1}{6(2n-3)!}$$

$$\Rightarrow \frac{1}{2n-2} = \frac{1}{(2n-1)(2n-2)} + \frac{1}{6}$$

$$\Rightarrow \frac{1}{2n-2} = \frac{1}{(2n-1)(2n-2)} + \frac{1}{6}$$

$$\Rightarrow \frac{1}{2n-2} = \frac{6 + (2n-1)(2n-2)}{6.(2n-1)(2n-2)}$$

$\Rightarrow \qquad 6(2n-1) = 6 + 4n^2 - 6n + 2$

$\Rightarrow \qquad 12n - 6 = 6 + 4n^2 - 6n + 2$

$\Rightarrow \qquad 4n^2 - 18n + 14 = 0$

$\Rightarrow \qquad 2n^2 - 9n + 7 = 0$

S59. In the binomial expansion of $(1 + x)^n$, the coefficients of the fifth, sixth and seventh terms are ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ respectively. Since ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.

$$\Rightarrow \qquad 2 \cdot {}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow \qquad 2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5} \qquad \text{[Dividing both sides by } {}^nC_5]$$

$$\Rightarrow \qquad 2 = \frac{\frac{n!}{4!(n-4)!}}{\frac{n!}{5!(n-5)!}} + \frac{\frac{n!}{6!(n-6)!}}{\frac{n!}{5!(n-5)!}}$$

$$2 = \frac{5!(n-5)!}{4!(n-4)!} + \frac{5!(n-5)!}{6!(n-6)!}$$

$$\Rightarrow \qquad 2 = \frac{5}{n-4} + \frac{n-5}{6} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \qquad 2 = \frac{30 + (n-4)(n-5)}{6(n-4)}$$

$\Rightarrow \qquad 12n - 48 = 30 + n^2 - 9n + 20$

$\Rightarrow \qquad n^2 - 21n + 98 = 0$

$\Rightarrow \qquad (n-14)(n-7) = 0$

$\Rightarrow \qquad n = 7, 14.$

S60. According to the question,

Coefficient of T_{r-1} : Coefficient of T_r : Coefficient of T_{r+1} = 1 : 3 : 5

$$\frac{{}^nC_{r-2}}{1} = \frac{{}^nC_{r-1}}{3} = \frac{{}^nC_r}{5} \qquad \dots \text{(i)}$$

If $\qquad \dfrac{{}^nC_{r-2}}{1} = \dfrac{{}^nC_{r-1}}{3} \quad \Rightarrow \quad 3\, {}^nC_{r-2} = {}^nC_{r-1}$

$$\Rightarrow \qquad 3.\frac{n!}{(r-2)!(n-r+2)!} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\Rightarrow \quad \frac{3}{(r-2)!(n-r+2)(n-r+1)!} = \frac{1}{(r-1)(r-2)!(n-r+1)!}$$

$$\Rightarrow \quad \frac{3}{n-r+2} = \frac{1}{r-1} \quad \Rightarrow \quad n - r + 2 = 3r - 3$$

$$\Rightarrow \quad n - 4r = -5 \qquad \dots \text{(ii)}$$

Again if $\frac{{}^nC_{r-1}}{3} = \frac{{}^nC_r}{5} \quad \Rightarrow \quad 5\ {}^nC_{r-1} = 3\ {}^nC_r$

$$\Rightarrow \quad 5.\frac{n!}{(r-1)!(n-r+1)!} = 3.\frac{n!}{r!(n-r)!}$$

$$\Rightarrow \quad \frac{5}{(r-1)!(n-r+1)(n-r)!} = \frac{3}{r(r-1)!(n-r)!}$$

$$\Rightarrow \quad \frac{5}{n-r+1} = \frac{3}{r} \quad \Rightarrow \quad 5r = 3n - 3r + 3$$

$$\Rightarrow \quad 3n - 8r = -3 \qquad \dots \text{(iii)}$$

Multiplying (ii) by 2 and subtracting from (iii), we get n = 7

Putting $n = 7$ in (ii), we get

$$7 - 4r = -5 \quad \Rightarrow \quad r = 3$$

Hence, $n = 7$ and $r = 3$.

S61. Let the three consecutive terms be r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms. Their coefficients in the expansion of $(1 + a)^n$ are ${}^nC_{r-1}$, ${}^nC_r$ and ${}^nC_{r+1}$ respectively.

According to the question :

$${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 1 : 7 : 42$$

$$\Rightarrow \quad \frac{{}^nC_{r-1}}{1} = \frac{{}^nC_r}{7} = \frac{{}^nC_{r+1}}{42}$$

If $\frac{{}^nC_{r-1}}{1} = \frac{{}^nC_r}{7} \quad \Rightarrow \quad 7\ {}^nC_{r-1} = {}^nC_r$ ... (i)

$$\Rightarrow \quad 7\frac{n!}{(r-1)!(n-r+1)!} = \frac{n!}{r(r-1)!(n-r)!}$$

$$\Rightarrow \quad \frac{7}{(r-1)!(n+1-r)(n-r)!} = \frac{1}{r(r-1)!(n-r)!}$$

$$\Rightarrow \quad \frac{7}{n+1-r} = \frac{1}{r}$$

$\Rightarrow \quad 7r = n + 1 - r$

$n = 8r - 1 \quad \dots \text{(ii)}$

Again, if $\quad \dfrac{{}^nC_r}{7} = \dfrac{{}^nC_{r+1}}{42}$

$\Rightarrow \quad 6 \cdot {}^nC_r = {}^nC_{r+1} \Rightarrow 6\,\dfrac{n!}{r!(n-r)!} = \dfrac{n!}{(r+1)!(n-r-1)!}$

$\Rightarrow \quad \dfrac{6}{r!(n-r)(n-r-1)!} = \dfrac{1}{(r+1)r!(n-r-1)!}$

$\Rightarrow \quad \dfrac{6}{n-r} = \dfrac{1}{r+1} = 6(r+1) = n - r$

$\Rightarrow \quad 6r + 6 = n - r \quad \Rightarrow \quad \text{n} = 7r + 6 \quad \dots \text{(iii)}$

From (ii) and (iii), we get

$8r - 1 = 7\text{ r} + 6 \quad \Rightarrow \quad r = 7$

Putting $r = 7$ in (iii), we get

$n = 7 \times 7 + 6 \quad \Rightarrow \quad n = 55$

S62. We have,

$T_2 = 240, \qquad T_3 = 720, \quad T_4 = 1080$

Now, $\quad T_2 = 240 \quad \Rightarrow \quad T_2 = {}^nC_1 x^{n-1}\, a = 240 \quad \dots \text{(i)}$

$T_3 = 720 \quad \Rightarrow \quad T_3 = {}^nC_2 x^{n-2}\, a^2 = 720 \quad \dots \text{(ii)}$

$T_4 = 1080 \quad \Rightarrow \quad T_4 = {}^nC_3 x^{n-3} a^3 = 1080 \quad \dots \text{(iii)}$

Dividing (ii) by (i), we get

$\dfrac{{}^nC_2 x^{n-2} a^2}{{}^nC_1 x^{n-1} a} = \dfrac{720}{240} \quad \Rightarrow \quad \dfrac{{}^nC_2}{{}^nC_1}\left(\dfrac{a}{x}\right) = \dfrac{3}{1}$

$\Rightarrow \quad {}^nC_2\left(\dfrac{a}{x}\right) = 3\,{}^nC_1 \quad \Rightarrow \quad \dfrac{n!}{2!(n-2)}\left(\dfrac{a}{x}\right) = 3\,\dfrac{n!}{1!(n-1)!}$

$\Rightarrow \quad \dfrac{a}{x(2!)(n-2)!} = \dfrac{3}{(n-1)(n-2)!} \quad \Rightarrow \quad \dfrac{a}{2x} = \dfrac{3}{n-1} \quad \dots \text{(iv)}$

Dividing (iii) by (ii), we get

$\dfrac{{}^nC_3 x^{n-3} a^3}{{}^nC_2 x^{n-2} a^2} = \dfrac{1080}{720} \quad \Rightarrow \quad \dfrac{{}^nC_3}{{}^nC_2}\left(\dfrac{a}{x}\right) = \dfrac{3}{2}$

$\Rightarrow \quad {}^nC_3\left(\dfrac{a}{x}\right) = \dfrac{3}{2}\,{}^nC_2 \quad \Rightarrow \quad \dfrac{n!}{(n-3)!3!}\left(\dfrac{a}{x}\right) = \dfrac{3}{2}\,\dfrac{n!}{2!(n-2)!}$

$$\Rightarrow \qquad \frac{1}{3(2!)(n-3)!}\left(\frac{a}{x}\right) = \frac{3}{2}\,\frac{1}{2!(n-2)(n-3)!}$$

$$\Rightarrow \qquad \frac{a}{3x} = \frac{3}{2(n-2)} \qquad \ldots \text{(v)}$$

From (iv) and (v), we have

$$\frac{a}{x} = \frac{6}{n-1} = \frac{9}{2(n-2)} \Rightarrow \qquad \frac{2}{n-1} = \frac{3}{2(n-2)}$$

$$\Rightarrow \qquad 4(n-2) = 3(n-1)$$

$$\Rightarrow \qquad 4n-8 = 3n-3 \qquad \Rightarrow \qquad n = 5$$

Putting $n = 5$ in (iv), we get $\frac{a}{2x} = \frac{3}{5-1}$ $\qquad \Rightarrow \qquad a = \frac{3x}{2}$

On putting the value of $n = 5$ and $a = \frac{3x}{2}$ in (i), we get

$$^5C_1 \cdot x^{5-1} \cdot \frac{3x}{2} = 240 \qquad \Rightarrow \qquad 5 \times \frac{3}{2} x^5 = 240 \Rightarrow x^5 = \frac{240 \times 2}{15}$$

$$x^5 = 32 \qquad \Rightarrow \qquad x = 2$$

But $\qquad a = \frac{3x}{2} = \frac{3 \times 2}{2} = 3$

Hence $\qquad n = 5,\ x = 2$ and $a = 3$

S63. The general term in the expansion of $\left(2 + \frac{x}{3}\right)^n$ is

$$t_{r+1} = {}^nC_r (2)^{n-r} \left(\frac{x}{3}\right)^r = {}^nC_r (2)^{n-r}\, 3^{-r}\, x^r \qquad \ldots \text{(i)}$$

Putting $r = 7$ in Eq. (i), we get

$$t_8 = {}^nC_7 (2)^{n-7} (3)^{-7} x^7.$$

$\therefore$ Coefficient of x^7 is ${}^nC_7 (2)^{n-7} (3)^{-7}$

Putting $r = 8$ in Eq. (i), we get

$$t_9 = {}^nC_8 (2)^{n-8} (3)^{-8} x^8.$$

$\therefore$ Coefficient of x^8 is ${}^nC_8 (2)^{n-8} (3)^{-8}$

Now, coefficient of x^7 = coefficient of x^8 (given)

$$\Rightarrow \qquad {}^nC_7 (2)^{n-7} (3)^{-7} = {}^nC_8 (2)^{n-8} (3)^{-8}$$

$$\frac{{}^nC_8}{{}^nC_7} = \frac{2^{n-7}\, 3^{-7}}{2^{n-8}\, 3^{-8}} \quad \Rightarrow \quad \frac{n!}{(n-8)!8!} \times \frac{(n-7)!7!}{n!} = 2 \times 3$$

$$\Rightarrow \quad \frac{(n-7)}{8} = 6 \quad \Rightarrow \quad n - 7 = 48 \quad \Rightarrow \quad n = 55.$$

S64. The coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^n$ are ${}^nC_1$, ${}^nC_2$ and ${}^nC_3$ respectively.

Now, these coefficients are in A.P.

$\Rightarrow$ ${}^nC_1$, ${}^nC_2$, ${}^nC_3$ are in A.P. $\Rightarrow$ $2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$$\Rightarrow \quad 2\left\{\frac{n!}{(n-2)!\,2!}\right\} = \frac{n!}{(n-1)!\,1!} + \frac{n!}{(n-3)!\,3!}$$

$$\Rightarrow \quad \frac{n!}{(n-3)!}\left(\frac{1}{n-2}\right) = \frac{n!}{(n-3)!}\left\{\frac{1}{(n-1)(n-2)} + \frac{1}{6}\right\}$$

$$\Rightarrow \quad \frac{1}{n-2} = \frac{1}{(n-1)(n-2)} + \frac{1}{6}$$

$$\frac{1}{n-2} = \frac{6 + (n-1)(n-2)}{6(n-1)(n-2)}$$

$$\Rightarrow \quad n^2 - 9n + 14 = 0.$$

$$\Rightarrow \quad (n-2)(n-7) = 0 \quad = \quad n = 7. \quad \left[\because\ n \neq 2 \text{ since in that case the expansion will not contain the } 4^{\text{th}} \text{ term.}\right]$$

S65. The general term in the expansion of $(1 + 3x - 2x^3)^{10}$ is

$$T = \frac{10!}{a!b!c!} \cdot (1)^a (3x)^b (-2x^3)^c \quad \text{where} \quad a + b + c = 10$$

$$T = \frac{10!}{a!b!c!} \cdot (3)^b (-2)^c x^{b+3c} \quad \text{where} \quad a + b + c = 10 \qquad \text{... (i)}$$

If this term contains x^7, then

$$b + 3c = 7 \qquad \text{... (ii)}$$

and $\quad a + b + c = 10 \qquad$ ... (iii)

From Eq. (ii), we get

$$c = \frac{7-b}{3} \qquad \text{... (iv)}$$

Since c can have only non-negative integral values, so the possible values of b are 1, 4 and 7 [Using (iv)]

When $b = 1$, we have: $c = 2$ and $a = 7$ [Using (ii) and (iii)]

When $b = 4$, we have: $c = 1$ and $a = 5$ [Using (ii) and (iii)]

When $b = 7$, we have: $c = 0$ and $a = 3$ [Using (ii) and (iii)]

Thus, there are three terms containing x^7. These terms are

$$\frac{10!}{7!1!2!}(3)^1(-2)^2x^7, \quad \frac{10!}{5!4!1!}(3)^4(-2)^1x^7, \quad \frac{10!}{3!7!0!}(3)^7(-2)^0x^7 \qquad \text{[Using (i)]}$$

The coefficient of x^7 in the expansion of $(1 + 3x - 2x^3)^{10}$

$$= \frac{10!}{7!1!2!}(3)^1(-2)^2 + \frac{10!}{5!4!1!}(3)^4(-2)^1 + \frac{10!}{3!7!0!}(3)^7(-2)^0$$

$$= 4320 - 204120 + 262440 = 62640.$$

Date: 10/10/2021

Q1. Find the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^6$.

Q2. Find the term independent of x in the expansion of $\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}$, $x > 0$.

Q3. Find the term independent of x in the expansion of $\left(x - \frac{1}{x}\right)^{12}$.

Q4. Find the term independent of x in the expansion of $\left(x^3 + \frac{3}{x^2}\right)^{15}$.

Q5. Find the term independent of x in the expansion of $\left(2x - \frac{1}{x}\right)^{10}$.

Q6. Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{x}\right)^9$.

Q7. Find the term independent of x in the expansion of $\left(x^2 + \frac{1}{2x}\right)^{12}$.

Q8. Find the term independent of x in the expansion of $\left(ax^2 + \frac{b}{x}\right)^{25}$.

Q9. Find the term independent of x in the expansion of $\left(ax + \frac{b}{x}\right)^{14}$.

Q10. Find the term independent of x in the expansion of $\left\{\sqrt{\frac{x}{3}} + \sqrt{\frac{3}{2x^2}}\right\}^{10}$.

Q11. Find the constant term in the expansion of $\left(x - \frac{1}{x}\right)^6$.

Q12. Find the term independent of x in the expansion of $\left\{\left(\frac{3x^2}{2}\right) - \left(\frac{1}{3x}\right)\right\}^9$.

Q13. Find the term independent of x in the expansion of $\left(\frac{\sqrt{x}}{3} + \frac{\sqrt{3}}{x^2}\right)^{10}$.

Q14. Find the term independent of x in the expansion of $\left(\frac{2\sqrt{x}}{5} - \frac{1}{2x\sqrt{x}}\right)^{11}$.

Q15. Find the term independent of x in the expansion of $\left(2x + \frac{1}{3x}\right)^6$.

Q16. Find the term independent of x in the expansion of $(1 + x)^n \left[1 + \left(\frac{1}{x}\right)\right]^n$.

Q17. Find the term independent of x in the expansion of $\left(ax^2 - \frac{b}{\sqrt{x}}\right)^{10}$.

Q18. Find the term independent of x in the expansion of $\left\{\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right\}^{10}$.

Q19. Find the coefficient of the term independent of x in the expansion of $\left\{\frac{(x+1)}{x^{2/3} - x^{1/3} + 1} - \frac{(x-1)}{x - x^{1/2}}\right\}^{10}$.

Date: 10/10/2021

S1. We have,
$$T_{r+1} = {}^{6}C_r \left(\frac{3}{2}x^2\right)^{6-r}\left(-\frac{1}{3x}\right)^r$$
$$= {}^{6}C_r \left(\frac{3}{2}\right)^{6-r}(x^2)^{6-r}(-1)^r\left(\frac{1}{x}\right)^r\left(\frac{1}{3^r}\right)$$
$$= (-1)^r\ {}^{6}C_r \frac{(3)^{6-2r}}{(2)^{6-r}}\cdot x^{12-3r}$$

The term will be independent of x if the index of x is zero. *i.e.*, $12 - 3r = 0$. Thus, $r = 4$.

Hence, 5th term is independent of x and is given by $(-1)^4\ {}^{6}C_4 \dfrac{(3)^{6-8}}{(2)^{6-4}} = \dfrac{5}{12}$.

S2. We have,
$$T_{r+1} = {}^{18}C_r \left(\sqrt[3]{x}\right)^{18-r}\left(\frac{1}{2\sqrt[3]{x}}\right)^r$$
$$= {}^{18}C_r\ x^{\frac{18-r}{3}}\cdot\frac{1}{2^r\cdot x^{\frac{r}{3}}} = {}^{18}C_r\ \frac{1}{2^r}\cdot x^{\frac{18-2r}{3}}$$

Since, we have to find a term independent of x, *i.e.*, term not having x, so take $\dfrac{18-2r}{3} = 0$.

We get $r = 9$. The required term is ${}^{18}C_9\ \dfrac{1}{2^9}$.(10th term is independent of x).

S3.
$$T_{r+1} = {}^{12}C_r x^{12-r}\left(-\frac{1}{x}\right)^r$$
$$= (-1)^r\cdot{}^{12}C_r\cdot x^{12-2r}$$

The term will be independent of x when $12 - 2r = 0$

$\Rightarrow$ $r = 6$

$\Rightarrow$ T_7 will be independent of x, $T_7 = (-1)^6\ {}^{12}C_6 = 924$

S4.
$$T_{r+1} = {}^{15}C_r (x^3)^{15-r}\left(\frac{3}{x^2}\right)^r$$
$$= {}^{15}C_r\cdot 3^r\cdot x^{45-5r}$$

The term will be independent of x when $45 - 5r = 0$

$\Rightarrow$ $45 - 5r = 0$

$\Rightarrow$ $r = 9$

$\Rightarrow$ T_{10} will be independent of x

$$T_{10} = {}^{15}C_9 \cdot 3^9 = (5005) \cdot 3^9$$

S5.

$$T_{r+1} = {}^{10}C_r (2x)^{10-r} \left(-\frac{1}{x}\right)^r$$

$$= (-1)^r\, 2^{10-r}.\ {}^{10}C_r\, x^{10-2r}$$

The term will be independent of x when $10 - 2r = 0$

$\Rightarrow$ $10 - 2r = 0$

$\Rightarrow$ $r = 5$

$\Rightarrow$ T_6 will be independent of x

$$T_6 = (-1)^5 \cdot 2^5 \cdot {}^{10}C_5 = -8064.$$

S6. Let $(r + 1)^{th}$ term be independent of x,

In the expansion $\left(x^2 + \frac{1}{x}\right)^9$ we have,

$$t_{r+1} = {}^9C_r (x^2)^{9-r} \left(\frac{1}{x}\right)^r$$

$$= {}^9C_r \cdot x^{18-3r}$$

Putting $18 - 3r = 0$ $\Rightarrow$ $r = 6$.

$\therefore$ $(6 + 1)^{th}$ term contains x^0.

$\therefore$ $t_{6+1} = {}^9C_r\, x^o = {}^9C_6 = 84.$

S7.

$$T_{r+1} = {}^{12}C_r (x^2)^{12-r} \left(\frac{1}{2x}\right)^r$$

$$= {}^{12}C_r \cdot \left(\frac{1}{2}\right)^r x^{24-3r}$$

The term will be independent of x when

$24 - 3r = 0$ $\Rightarrow$ $r = 8$

$\Rightarrow$ T_9 will be independent of x

$$T_9 = {}^{12}C_8 \left(\frac{1}{2}\right)^8 = \frac{495}{256}.$$

S8. The general term in the expansion of $\left(ax^2 + \frac{b}{x}\right)^{25}$ is

$$t_{r+1} = {}^{25}C_r\,(ax^2)^{25-r}\left(\frac{b}{x}\right)^r$$

$$= {}^{25}C_r\,a^{25-r}\,b^r\,x^{50-3r}$$

Now, this term will be independent of *x* if

$$50 - 3r = 0 \quad i.e., \quad \text{if } r = \frac{50}{3} \notin I.$$

Clearly, there is no term in the expansion of $\left(ax^2 + \frac{b}{x}\right)^{25}$ which is independent of *x*.

S9. The general term in the expansion of $\left(ax + \frac{b}{x}\right)^{14}$ is

$$t_{r+1} = {}^{14}C_r\,(ax)^{14-r}\left(\frac{b}{x}\right)^r = {}^{14}C_r\,a^{14-r}\,b^r\,x^{14-2r} \qquad \dots \text{(i)}$$

Now, this term will be independent of *x* if $14 - 2r = 0$, *i.e.*, if $r = 7$.

Putting $r = 7$ in Eq. (i), we get the term independent of *x*, as:

$$t_8 = {}^{14}C_7\,a^7\,b^7\,x^0 = \frac{14!}{(7!)^2}\,a^7\,b^7.$$

S10. The general term in the expansion of $\left\{\sqrt{\frac{x}{3}} + \sqrt{\frac{3}{2x^2}}\right\}^{10}$ is

$$t_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\sqrt{\frac{3}{2x^2}}\right)^r$$

$$\Rightarrow \qquad t_{r+1} = {}^{10}C_r\,(3)^{r-5}\,(2)^{-\frac{r}{2}}\,(x)^{5-\frac{3r}{2}} \qquad \dots \text{(i)}$$

Now, this term will be independent of *x* if $5 - \frac{3r}{2} = 0$ *i.e.*, $r = \frac{10}{3}$ which is not possible since *r* must be a non-negative integer.

Hence, there is no term in the given expansion, which is independent of *x*.

S11. The general term in the expansion of $\left(x - \frac{1}{x}\right)^6$ is

$$t_{r+1} = (-1)^r\,{}^6C_r\,x^{6-r}\left(\frac{1}{x}\right)^r = (-1)^r\,{}^6C_r\,x^{6-2r} \qquad \dots \text{(i)}$$

If this term is a constant term *i.e.*, if it is independent of *x*, then

$$6 - 2r = 0 \quad i.e., \quad r = 3$$

$\therefore$ The constant term in the expansion of $\left(x - \frac{1}{x}\right)^6$ is

$$t_4 = (-1)^3\, {}^6C_3\, x^0 \qquad \text{[Putting } r = 3 \text{ in Eq. (i)]}$$

i.e., $$t_4 = -\left(\frac{6 \times 5 \times 4}{3 \times 2}\right) = -\mathbf{20}.$$

S12. The general term in the expansion of $\left\{\left(\frac{3x^2}{2}\right) - \left(\frac{1}{3x}\right)\right\}^9$ is

$$t_{r+1} = (-1)^r\, {}^9C_r \left(\frac{3x^2}{2}\right)^{9-r} \left(\frac{1}{3x}\right)^r$$

$$= (-1)^r\, {}^9C_r\, (3)^{9-2r} \cdot (2)^{r-9}\, (x)^{18-3r} \qquad \text{... (i)}$$

This term will be independent of x if

$$18 - 3r = 0 \quad i.e., \quad \text{if } r = 6.$$

$\therefore$ The term independent of x in the expansion of $\left\{\left(\frac{3x^2}{2}\right) - \left(\frac{1}{3x}\right)\right\}^9$ is

$$t_7 = (-1)^6\, {}^9C_6\, (3)^{-3}\, (2)^{-3}\, x^0 \qquad \text{[Putting } r = 6 \text{ in Eq. (i)]}$$

$$= \frac{9 \times 8 \times 7}{3 \times 2} \times \frac{1}{3^3 \times 2^3} = \frac{7}{18}.$$

S13. The general term in the expansion of $\left(\frac{\sqrt{x}}{3} + \frac{\sqrt{3}}{x^2}\right)^{10}$ is

$$t_{r+1} = {}^{10}C_r \left(\frac{\sqrt{x}}{3}\right)^{10-r} \left(\frac{\sqrt{3}}{x^2}\right)^r$$

$$= {}^{10}C_r\, (3)^{r - 10 + \frac{r}{2}}\, (x)^{5 - \frac{r}{2} - 2r} \qquad \text{... (i)}$$

Now, this term will be independent of x if $5 - \frac{r}{2} - 2r = 0$ *i.e.*, if $5 - \frac{5r}{2} = 0$ *i.e.*, if $r = 2$.

$\therefore$ The term independent of x in the expansion of $\left(\frac{\sqrt{x}}{3} + \frac{\sqrt{3}}{x^2}\right)^{10}$ is

$$t_3 = {}^{10}C_2 (3)^{2-10+1} x^0 = {}^{10}C_2 \left(\frac{1}{3^7}\right)$$ [Putting $r = 2$ in Eq. (i)]

S14. The general term in the expansion of $\left(\frac{2\sqrt{x}}{5} - \frac{1}{2x\sqrt{x}}\right)^{11}$ is

$$t_{r+1} = {}^{11}C_r (-1)^r \left(\frac{2\sqrt{x}}{5}\right)^{11-r} \left(\frac{1}{2x\sqrt{x}}\right)^r$$

$\Rightarrow$ $$t_{r+1} = {}^{11}C_r (-1)^r 2^{11-2r} 5^{r-11} (x)^{\frac{1}{2}(11-r)-\frac{3}{2}r}$$

$\Rightarrow$ $$t_{r+1} = {}^{11}C_r (-1)^r 2^{11-2r} 5^{r-11} (x)^{\frac{11}{2}-2r} \quad \text{... (i)}$$

This term will be independent of x if $\frac{11}{2} - 2r = 0$ *i.e.*, if $r = \frac{11}{4}$.

Since $\frac{11}{4}$ is not an integer, so there exists no term which is independent of x.

Hence, no term is independent of x as $r \notin I$.

S15. The general term in the expansion of $\left(2x + \frac{1}{3x}\right)^6$ is

$$t_{r+1} = {}^6C_r (2x)^{6-r} \left(\frac{1}{3x}\right)^r = {}^6C_r (2)^{6-r} (3)^{-r} x^{6-2r} \quad \text{... (i)}$$

This term will be independent of x if $6 - 2r = 0$, *i.e.*, $r = 3$.

$\therefore$ The term independent of x in the expansion of $\left(2x + \frac{1}{3x}\right)^6$ is

$$t_{r+1} = {}^6C_3 (2)^3 (3)^{-3} x^0$$ [Putting $r = 3$ in Eq. (i)]

$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{8}{27} = \frac{160}{27}.$$

S16. By Binomial Theorem, we have:

$$(1 + x)^n \left[1 + \left(\frac{1}{x}\right)\right]^n = \{{}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n\}$$

$$\times \left\{{}^nC_0 + {}^nC_1\left(\frac{1}{x}\right) + {}^nC_2\left(\frac{1}{x}\right)^2 + \dots + {}^nC_n\left(\frac{1}{x}\right)^n\right\}$$

$\therefore$ The term independent of x in this expansion

$$= {}^nC_0 \cdot {}^nC_0 + {}^nC_1 \cdot {}^nC_1 + {}^nC_2 \cdot {}^nC_2 + \dots + {}^nC_n \cdot {}^nC_n$$

$$= ({}^nC_0)^2 + ({}^nC_1)^2 + ({}^nC_2)^2 + \dots + ({}^nC_n)^2$$

$$= C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2.$$

S17. Let t_{r+1} be the term independent of x in the expansion of $\left(ax^2 - \frac{b}{\sqrt{x}}\right)^{10}$.

We have: $$t_{r+1} = (-1)^r \cdot {}^{10}C_r\,(ax^2)^{10-r}\left(\frac{b}{\sqrt{x}}\right)^r$$

$\Rightarrow$ $$t_{r+1} = (-1)^r \cdot {}^{10}C_r \cdot a^{10-r}\, b^r\, x^{\left(20-\frac{5r}{2}\right)} \quad \dots \text{(i)}$$

Putting $20 - \frac{5r}{2} = 0$, we get: $r = 8$

Putting $r = 8$ in Eq. (i), we get:

$$t_9 = (-1)^8 \cdot {}^{10}C_8 \cdot a^2\, b^8\, x^0 = {}^{10}C_8\, a^2\, b^8$$

Clearly, the 9th term of the expansion is independent of x.

S18. The general term in the expansion of $\left\{\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right\}^{10}$ is

$$t_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r$$

$$= {}^{10}C_r\,(3)^{\frac{3}{2}r-5}(2)^{-r}(x)^{5-\frac{5}{2}r} \quad \dots \text{(i)}$$

This term will be independent of x if $5 - \frac{5}{2}r = 0$ *i.e.*, if $r = 2$.

$\therefore$ The term independent of x in the expansion is

$$t_3 = {}^{10}C_2\,(3)^{-2}(2)^{-2}x^0 \quad \text{[Putting } r = 2 \text{ in Eq. (i)]}$$

$$= \frac{10 \times 9}{2} \times \frac{1}{3^2 \times 2^2} = \frac{5}{4}.$$

S19. Let, $$S = \left\{\frac{(x+1)}{x^{2/3} - x^{1/3} + 1} - \frac{(x-1)}{x - x^{1/2}}\right\}^{10}$$

$$= \left\{\frac{(x^{1/3}+1)(x^{2/3} - x^{1/3} + 1)}{(x^{2/3} - x^{1/3} + 1)} - \frac{(x - x^{1/2})(x + x^{1/2})}{x\,(x - x^{1/2})}\right\}^{10}$$

$$= \left\{(x^{1/3}+1)-\left(\frac{x+x^{1/2}}{x}\right)\right\}^{10} = \left\{(x^{1/3}+1)-\left(1+\frac{1}{x^{1/2}}\right)\right\}^{10}$$

$$= \left\{x^{1/3}-\frac{1}{x^{1/2}}\right\}^{10}$$

Now, the general term in the expansion of $\left\{x^{1/3}-\frac{1}{x^{1/2}}\right\}^{10}$ is

$$t_{r+1} = {}^{10}C_r\,(-1)^r\,(x^{1/3})^{10-r}\left(\frac{1}{x^{1/2}}\right)^r$$

$$= {}^{10}C_r\,(-1)^r\,x^{\frac{10}{3}-\frac{5}{6}r} \qquad \dots \text{(i)}$$

This term is independent of x if $\frac{10}{3}-\frac{5}{6}r = 0$, *i.e.*, if $r = 4$

Putting $r = 4$ in Eq. (i), we get

$$t_5 = {}^{10}C_4\,(-1)^4\,x^0 = {}^{10}C_4$$

$$= \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} = \mathbf{210}.$$

Hence, the coefficient of the term independent of x is 210.

Date: 10/10/2021

Q1. **Find the greatest coefficient in the expansion of $(1 + x)^{2n}$.**

Q2. **Find the greatest term in the expansion of $(2 + 3x)^9$, when $x = 3/2$.**

Q3. **Find the greatest term in the expansion of $(1 + 4x)^8$, when $x = \frac{1}{3}$.**

Q4. **Find the numerically greatest term in the expansion of $(3 + 2x)^{44}$ when $x = \frac{1}{5}$.**

Date: 10/10/2021

S1. Since $2n$ is an even number, so the greatest coefficient in the expansion of $(1 + x)^{2n}$ is the coefficient of the middle term *i.e.*, of the $\left(\frac{2n}{2}+1\right)^{th}$ term *i.e.*, of the $(n + 1)^{th}$ term i.e. ${}^{2n}C_n$.

S2. Let,

$$(2 + 3x)^9 = 2^9\left[1+\frac{3x}{2}\right]^9$$

In the expansion of $(1 + x)^n$.

$$\frac{T_{r+1}}{T_r} = \frac{{}^nC_r x^r}{{}^nC_{r-1}x^{r-1}} = \frac{n-r+1}{r}\cdot x$$

Here, $$n = 9,\ x = \frac{3}{2}\cdot\frac{3}{2} = \frac{9}{4}$$

Now, $$T_{r+1} \geq T_r$$

$\Rightarrow$ $$T_{r+1} \geq T_r$$

$\Rightarrow$ $$\frac{9(10-r)}{4r} \geq 1 \qquad \Rightarrow \qquad 90 - 9r \geq 4r$$

$\Rightarrow$ $$r \leq \frac{90}{13}, \qquad \Rightarrow \quad r = 6$$

$\Rightarrow$ T_7 is the greatest term

$$T_7 = 2^9\cdot{}^9c_6\left[\frac{3}{2}\cdot\frac{3}{2}\right]^6 = \frac{7}{2}\cdot 3^{13}.$$

S3. In the expansion of $(1 + x)^n$.

$$\frac{T_{r+1}}{T_r} = \frac{{}^nC_r x^r}{C_{r-1}x^{r-1}}$$

$$= \frac{\frac{n!}{r!(n-r)!}x^r}{\frac{n!}{(r-1)!(n-r+1)!}x^{r-1}}$$

$$= \frac{n-r+1}{r}x$$

Here, $$n = 8, \quad x = 4\cdot\frac{1}{3}$$

Now, $$T_{r+1} \geq T_r$$

$$\Rightarrow \quad \frac{4(9-r)}{3r} \geq 1 \quad \Rightarrow \quad 36 - 4r \geq 3r$$

$$\Rightarrow \quad r \geq \frac{36}{7} \quad \Rightarrow \quad r = 5$$

$\Rightarrow$ T_6 is the greatest term.

$$T_6 = {}^8C_5 \left(\frac{4}{3}\right)^5 = 56 \left(\frac{4}{3}\right)^5.$$

S4. The general term in the expansion $(3 + 2x)^{44}$ is

$$t_{r+1} = {}^{44}C_r\,(3)^{44-r}(2x)^r$$

$$\Rightarrow \quad t_r = {}^{44}C_{r-1}(33)^{44-(r-1)}\,(2x)^{r-1} = {}^{44}C_{r-1}\,(3)^{45-r}(2x)^{r-1}$$

So, $$\frac{t_{r+1}}{t_r} = \frac{{}^{44}C_r(3)^{44-r}(2x)^r}{{}^{44}C_{r-1}(3)^{45-r}(2x)^{r-1}} = \frac{45-r}{r} \cdot \frac{2x}{3}$$

When $$x = \frac{1}{5}, \quad \text{we have: } \frac{t_{r+1}}{t_r} = \frac{45-r}{r} \cdot \frac{2}{3} \cdot \frac{1}{5} = \frac{2(45-r)}{15r}$$

Now, $$\frac{t_{r+1}}{t_r} > 1 \Rightarrow \frac{2(45-r)}{15r} > 1 \Rightarrow 17r < 90 \Rightarrow r < \frac{90}{17} = 5\frac{5}{7}$$

Clearly, $$\frac{t_{r+1}}{t_r} > 1 \text{ i.e., } t_{r+1} > t_r \text{ for all values of } r \leq 5.$$

Hence, t_6 is numerically greatest term.

Date: 10/10/2021

Q1. Compute $(98)^5$.

Q2. If a and b are distinct integers, prove that $a - b$ is a factor of $a^n - b^n$, whenever n is a positive integer.

Q3. Indicate which number is larger (use binomial theorem to explain your answer): $(1.2)^{4000}$ or 800.

Q4. Using binomial theorem, compute the following $(99)^5$.

Q5. Using binomial theorem, compute the following $(98)^4$.

Q6. Using Binomial Theorem, evaluate: $(999)^5$

Q7. Using Binomial Theorem, evaluate: $(102)^5$

Q8. Which number is larger : $(1.1)^{10000}$ or 1000.

Q9. Write down the binomial expansion of $(1 + x)^{n+1}$, when $x = 8$. Deduce that $9^{n+1} - 8n - 9$ is divisible by 64, where n is a positive integer.

Q10. Using binomial theorem evaluate $(103)^6$.

Q11. Indicate which number is larger (Use Binomial Theorem to explain your answer): $(1.01)^{1000000}$ or 10,000

Q12. Using binomial theorem, prove that $2^{3n} - 7n - 1$ is divisible by 49, where $n \in N$.

Q13. Using Binomial Theorem, prove that $6^n - 5n$ always leaves reminder 1 when divided by 25.

Q14. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

Q15. Using Binomial Theorem, evaluate the following: $(96)^3$.

Q16. Using Binomial Theorem, evaluate the following: $(101)^4$.

Date: 10/10/2021

S1. We express 98 as the sum or difference of two numbers whose powers are easier to calculate, and then use Binomial Theorem.

Write $98 = 100 - 2$

Therefore, $(98)^5 = (100-2)^5$

$= {}^5C_0\,(100^5(2)^0 - {}^5C_1\,(100)^4 \cdot 2 + {}^5C_2\,(100)^3 2^2$

$- {}^5C_3\,(100)^2(2)^3 + {}^5C_4\,(100)\,(2)^4 - {}^5C_5\,(2)^5$

$= 10000000000 - 5 \times 100000000 \times 2 + 10 \times 1000000 \times 4$

$- 10 \times 10000 \times 8 + 5 \times 100 \times 16 - 32$

$= 10040008000 - 1000800032 = 9039207968.$

S2. We have, $a^n - b^n = {}^nC_1\, b^{n-1}(a-b) + {}^nC_2\, b^{n-2}(a-b)^2 + \ldots + (a-b)^n$

$= (a-b)\,[{}^nC_1\, b^{n-1}(a-b) + {}^nC_2\, b^{n-2}(a-b) + \ldots (a-b)^{n-1}]$

Thus, $(a-b)$ is a factor of $(a^n - b^n)$.

S3. $(1.2)^{4000} = (1+0.2)^{4000} = 1 + {}^{4000}C_1(0.2) + {}^{4000}C_2(0.2)^2 + \ldots$

$= 1 + 4000(0.2) + \ldots \quad = 1 + 800 + {}^{4000}C_2(0.2)^2 + \ldots \; > 800$

Hence $(1.2)^{4000} > 800$

S4. $(100-1)^5 = {}^5C_0(100)^5 - {}^5C_1(100)^4 + {}^5C_2(100)^3 - {}^5C_3(100)^2 + {}^5C_4(100) - {}^5C_5(100)^0$

$= (100)^5 - 5\,(100)^4 + 10\,(100)^3 - 10(100)^2 + 5 \times 100 - 1$

$= 10^{10} - 5 \times 10^8 + 10^7 - 10^5 + 5 \times 10^2 - 1$

$= (10^{10} + 10^7 + 5 \times 10^2) - (5 \times 10^8 + 10^5 + 1)$

$= 10010000500 - 500100001 = 9509900499$

S5. $(100-2)^4 = {}^4C_0(100)^4 - {}^4C_1\,(100)^3\,2^1 + {}^4C_2\,(100)^2\,2^2 - {}^4C_3\,(100)(2)^3 + {}^4C_4 2^4$

$= 100000000 - 4 \times 1000000 \times 2 + 6 \times 10000 \times 4 - 4 \times 100 \times 8 + 16$

$= 100000000 - 8000000 + 240000 - 3200 + 16 = 92236816$

S6. We have, $(999)^5 = (1000-1)^5 = (10^3 - 1)^5$

Expanding by binomial theorem, we get

$(999)^5 = (10^3-1)^5 \quad = {}^5C_0(10^3)^5 - {}^5C_1(10^3)^4(1) + {}^5C_2(10^3)^3(1)^2 - {}^5C_3\,(10^3)^2(1)^3$

$+ {}^5C_4\,(10^3)(1)^4 - {}^5C_5(1)^5$

$= (10)^{15} - 5 \times 10^{12} + 10(10)^9 - 10(10)^6 + 5(10)^3 - 1$

$= 10^{10}[10^5 - 5 \times 100 + 1] - 10^3\,[10^4 - 5] - 1$

$= 10^{10}[100000 - 500 + 1] - 10^3[10000 - 5] - 1$

$= 99501 \times 10^{10} - 9995 \times 10^3 = 975009990004$

S7. $(102)^5 = (100 + 2)^5$

$$= {}^5C_0 \times (100)^5 + {}^5C_1 \times (100)^4 \times 2 + {}^5C_2 \times (100)^3 \times 2^2 + {}^5C_3 \times (100)^2 \times 2^3 + {}^5C_4 \times (100)^1 \times 2^4 + {}^5C_5 \times (100)^0 \times 2^5$$

$$= (100)^5 + 5 \times (100)^4 \times 2 + 10 \times (100)^3 \times 2^2 + 10 \times (100)^2 \times 2^3 + 5 \times (100)^1 \times 2^4 + (100)^0 \times 2^5$$

$$= 10^{10} + 10^9 + 4 \times 10^7 + 8 \times 10^5 + 8 \times 10^3 + 32$$

$$= 11040808032$$

S8. We have,

$$(1.1)^{1000} = [1 + (0.1)]^{10000}$$

Expanding by binomial theorem,

$$= {}^{10000}C_0 (1)^{10000} + {}^{10000}C_1 (1)^{10000-1}(0.1) + {}^{10000}C_2 (1)^{10000-2}(0.1)^2 + \ldots$$

$$= 1 + 10000 \times (0.1) + \text{other terms}$$

$$= 1001 + \text{other terms of the expansion}$$

Hence $(1.1)^{1000} > 1000$

S9. We have,

$$(1 + x)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1 x + {}^{n+1}C_2 x^2 + \ldots + {}^{n+1}C_{n+1} x^{n+1}$$

putting $x = 8$, we get

$$(1 + 8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8)^1 + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$$

$\Rightarrow$ $9^{n+1} = 1 + (n + 1) \times 8 + {}^{n+1}C_2(8)^2 + {}^{n+1}C_3(8)^3 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$

$\Rightarrow$ $9^{n+1} - 8n - 9 = (8)^2[{}^{n+1}C_2 + {}^{n+1}C_3(8) + \ldots + {}^{n+1}C_{n+1}(8)^{n-1}]$

$\Rightarrow$ $9^{n+1} - 8n - 9 = 64 \times$ an integer.

$\Rightarrow$ $9^{n+1} - 8n - 9$ is divisible by 64.

S10. We have, $(103)^6$

Expanding by binomial theorem, we get

$$(103)^6 = (10^2 + 3)^6$$

$$= {}^6C_0(10^2)^6 + {}^6C_1(10^2)^5(3)^1 + {}^6C_2(10^2)^4(3)^2 + {}^6C_3(10^2)^3(3)^3 + {}^6C_4(10^2)^2(3)^4 + {}^6C_5(10^2)(3)^5 + {}^6C_6(3)^6.$$

$$= (10^{12}) + 6(10^{10})(3) + 15(10^8)(9) + 20(10^6)(27) + 15(10^4)(81) + 6(10^2)(243) + 729$$

$$= 10^{12} + 18(10^{10}) + 135(10^8) + 540(10^6) + 1215(10^4) + 1458(10^2) + 729 = 1194052296529.$$

S11. $(1 + 0.01)^{1000000} = 1 + {}^{1000000}C_1(0.01) + {}^{1000000}C_2(0.01)^2 + ...$

$= 1 + 1000000(0.01) + {}^{1000000}C_2(0.01)^2 + ... = 1 + 10,000$

$+ 1000000C_2(0.01)^2 + ... > 1000$

$\therefore \quad (1.01)^{1000000} > 10,000$

S12. $2^{3n} - 7n - 1 = 8^n - 7n - 1$

$(1 + 7)^n = 1 + {}^nC_1 7 + {}^nC_2 7^2 + {}^nC_3 7^3 + ...$

$8^n = 1 + 7n + 7^2 [{}^nC_2 + {}^nC_3 7 + ...]$

$8n - 7n - 1 = 49$ [An integer]

$\therefore \quad 8^n - 7n - 1$ is divisible by 49, $n \in N$

or $2^{3n} - 7n - 1$ is divisible by 49, $n \in N$

S13. For two numbers a and b if we can find numbers q and r such that $a = bq + r$, then we say that b divides a with q as quotient and r as reminder. Thus, in order to show that $6^n - 5n$ leaves reminder 1 when divided by 25, we prove that $6^n - 5n = 25k + 1$, where k is some natural number.

We have,

$$(1 + a)^n = {}^nC_0 + {}^nC_1 a + {}^nC_2 a^2 + ... + {}^nC_n a^n$$

For $a = 5$, we get

$$(1 + 5)^n = {}^nC_0 + {}^nC_1 5 + {}^nC_2 5^2 + ... + {}^nC_n 5^n$$

i.e., $(6)^n = 1 + 5n + 5^2 . {}^nC_2 + 5^3 . {}^nC_3 + 5^n$

i.e., $6^n - 5n = 1 + 5^2 ({}^nC_2 + {}^nC_3 5 + ... + 5^{n-2})$

or $6^n - 5n = 1 + 25 ({}^nC_2 + 5 . {}^nC_3 + ... + 5^{n-2})$

or $6^n - 5n = 25k + 1$

where $k = {}^nC_2 + 5 . {}^nC_3 + ... + 5^{n-2}$

This shows that when divided by 25, $6^n - 5n$ leaves remainder 1.

S14. We have, $(0.99)^5 = (1 - 0.01)^5$

$$= \left(1 - \frac{1}{100}\right)^5$$

$$= {}^5C_0 - {}^5C_1 \times \frac{1}{100} + {}^5C_2 \times \left(\frac{1}{100}\right)^2 - {}^5C_3\left(\frac{1}{100}\right)^3 + {}^5C_4\left(\frac{1}{100}\right)^4 - {}^5C_5\left(\frac{1}{100}\right)^5$$

$$= 1 - \frac{5}{100} + \frac{10}{10000} - \frac{10}{1000000} + \frac{5}{(100)^4} - \frac{1}{(100)^5}$$

$= 1 - 0.05 + 0.001$ [Neglecting fourth and other terms]

$= 0.951.$

S15. We express 96 as the sum or difference of two numbers whose powers are easier to calculate, and then use Binomial Theorem

Write, $96 = 100 - 4$

Therefore,

$$(96)^3 = (100 - 4)^3$$
$$= {}^3C_0\,(100)^3 - {}^3C_1\,(100)^2\,(4) + {}^3C_2\,(100)^1(4)^2 - {}^3C_3\,(4)^3$$
$$= 1000000 - 3\,(10000)\,(4) + 3\,(100)\,(16) - (64)$$
$$= 1000000 - 120000 + 4800 - 64$$
$$= 884736.$$

S16. We express 101 as the sum or difference of two numbers whose powers are easier to calculate, and then use Binomial Theorem

Write, $101 = 100 + 1$

Therefore,

$$(101)^4 = (100 + 1)^4$$
$$= {}^4C_0\,(100)^4 - {}^4C_1\,(100)^3\,(1) + {}^4C_2\,(100)^2(1)^2 + {}^4C_3\,(100)^1(1)^3 + {}^4C_4\,(1)$$
$$= 100000000 + 4\,(1000000) + 6\,(10000) + 4\,(100) + 1$$
$$= 100000000 + 4000000 + 60000 + 400 + 1$$
$$= 104060401.$$

Date: 10/10/2021

Q1. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that $c_0 + c_1 + c_2 + \ldots + c_n = 2^n$.

Q2. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0 + \frac{c_1}{2} + \frac{c_2}{3} + \ldots + \frac{c_n}{n+1} = \frac{2^{n+1} - 1}{n+1}.$$

Q3. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$\frac{1}{1!(n-1)!} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \ldots = \frac{2^{n-1}}{n!}.$$

Q4. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$(c_0 + c_2 + c_4 + \ldots) = (c_1 + c_3 + c_5 + \ldots) = 2^{n-1}.$$

Q5. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_1 + 2c_2 + 3c_3 + \ldots + nc_n = n\, 2^{n-1}.$$

Q6. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0 - \frac{c_1}{2} + \frac{c_2}{3} - \frac{c_3}{4} + \ldots + (-1)^n \frac{c_n}{n+1} = \frac{1}{n+1}$$

Q7. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0c_2 + c_1c_3 + c_2c_4 + \ldots + c_{n-2}\, c_n = \frac{(2n)!}{(n-2)!(n+2)!}.$$

Q8. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + \ldots + n\frac{c_n}{c_{n-1}} = \frac{n(n+1)}{2}.$$

Q9. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0c_r + c_1c_{r+1} + c_2c_{r+2} + c_3c_{r+3} + \ldots + c_{n-r}\,c_n = \frac{(2n)!}{(n-r)!(n+r)!}$$

Q10. If $c_0, c_1, c_2, \ldots, c_n$ denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0c_1 + c_1c_2 + c_2c_3 + \ldots + c_{n-1}c_n = \frac{n \cdot 2^n[1 \cdot 3 \cdot 5 \ldots (2n-1)]}{(n-1)!(n+1)!}.$$

Q11. If c_0, c_1, c_2, ..., c_n denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$2c_0 + 2^2\frac{c_1}{2} + 2^3\frac{c_2}{3} + 2^4\frac{c_3}{4} + \ldots + 2^{n+1}\frac{c_n}{n+1} = \frac{3^{n+1} - 1}{n+1}.$$

Q12. If c_0, c_1, c_2, ..., c_n denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0^2 + c_1^2 + c_2^2 + \ldots + c_n^2 = \frac{(2n)!}{(n!)^2}$$

Q13. If c_0, c_1, c_2, ..., c_n denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0 + 3c_1 + 5c_2 + \ldots + (2n + 1)c_n = (n + 1) \cdot 2^n$$

Q14. If c_0, c_1, c_2, ..., c_n denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$c_0 + 2c_1 + 3c_2 + \ldots + (n + 1)c_n = (n + 2) \cdot 2^{n-1}$$

Q15. If c_0, c_1, c_2, ..., c_n denote the coefficients of the successive terms in the expansion of $(1 + x)^n$, prove that

$$(c_0 + c_1)(c_1 + c_2)(c_2 + c_3)\ldots(c_{n-1} + c_n) = \frac{c_0c_1 \ldots c_{n-1} \cdot (n+1)^n}{n!}.$$

Date: 10/10/2021

S1. $$(1 + x)^n = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$$

put $$x = 1$$

$$2^n = c_0 + c_1 + c_2 + \dots + c_n.$$

S2. Given, $$c_0 + \frac{c_1}{2} + \frac{c_2}{3} + \dots + \frac{c_n}{n+1} = 1 + \frac{n}{2} + \frac{n(n-1)}{3.2} + \dots + \frac{1}{n+1}$$

$$= \frac{1}{n+1}\left[(n+1) + \frac{(n+1)n}{2!} + \frac{(n+1)n(n-1)}{3!} + \dots + 1\right]$$

$$= \frac{1}{n+1}\left[{}^{n+1}c_1 + {}^{n+1}c_2 + {}^{n+1}c_3 + \dots + {}^{n+1}c_{n+1}\right] = \frac{1}{n+1}[2^{n+1} - 1]$$

S3. $$\frac{1}{1!(n-1)} + \frac{1}{3!(n-3)!} + \frac{1}{5!(n-5)!} + \dots$$

$$= \frac{1}{n!}\left[\frac{n!}{1!(n-1)!} + \frac{n!}{3!(n-3)!} + \frac{n!}{5!(n-5)!} + \dots\right]$$

$$= \frac{1}{n!}\left[n + \frac{n(n-1)(n-2)}{3!} + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!} + \dots\right]$$

$$= \frac{1}{n!}[c_1 + c_3 + c_5 + \dots] = \frac{1}{n!}\cdot 2^{n-1}.$$

S4. $$(1 + x)^n = c_0 + c_1x + c_2x^2 + \dots + c_nx_n$$

put $$x = -1$$

$$0 = c_0 - c_1 - c_2 - c_3 + c_4 + \dots$$

$\Rightarrow$ $$c_1 + c_3 + c_5 + \dots = c_0 + c_2 + c_4 \dots$$

We know that

$$c_0 + c_1 + c_2 + c_3 + \dots + c_n = 2^n$$

$\Rightarrow$ $$[c_0 + c_2 + c_4 + \dots] = 2^n$$

$\Rightarrow$ $$c_0 + c_2 + c_4 + \dots = 2^{n-1}$$

and $$c_1 + c_3 + c_5 + \dots = 2^{n-1}$$

S5. $c_1 + 2c_2 + 3c_3 + \dots + nc_n$

$$= n + 2\frac{n(n-1)}{2!} + 3n\frac{(n-1)(n-2)}{3!} + ... + n \cdot 1$$

$$= n\left[1 + (n-1) + \frac{(n-1)(n-2)}{2!} + ... + 1\right]$$

$$= n[{}^{n-1}c_0 + {}^{n-1}c_1 + {}^{n-1}c_2 + ... + {}^{n-1}c_{n-1}]$$

$$= n \cdot 2^{n-1}$$

S6. $$c_0 - \frac{c_1}{2} + \frac{c_2}{3} - \frac{c_3}{4} + ... + (-1)^n \frac{c_n}{n+1} = 1 - \frac{n}{2} + \frac{n(n-1)}{3.2} - ... + \frac{(-1)^n}{n+1}$$

$$= \frac{1}{n+1}\left[(n+1) - \frac{(n+1)(n)}{2!} + \frac{(n+1)(n)(n-1)}{3!} - ... + (-1)^n\right]$$

$$= \frac{1}{n+1}[1 - \{{}^{n+1}c_0 - {}^{n+1}c_1 + {}^{n+1}c_2 + ... + (-1)^n\ {}^{n+1}c_{n+1}\}]$$

$$= \frac{1}{n+1}[1 - 0] = \frac{1}{n+1}.$$

S7. We know that

$$(1 + x)^{2n} = (1 + x)^n(x + 1)^n$$

$$\Rightarrow \quad {}^{2n}c_0 + {}^{2n}c_1 x + {}^{2n}c_2 x^2 + ... + {}^{2n}c_r x^r + ... + {}^{2n}c_{2n} x^{2n}$$

$$= [c_0 + c_1 x + c_2 x^2 + ... + c_n x^n]\ [c_0 x^n + c_1 x^{n-1} + c_2 x^{n-2} + ... + c_n]$$

Comparing the coefficient of x^{n-2}, we get

$${}^{2n}c_{n-2} = c_0c_2 + c_1c_3 + c_2c_4 + ... + c_{n-2}c_n$$

$$\Rightarrow \quad \frac{(2n)!}{(n-2)!(n+2)!} = c_0c_2 + c_1c_3 + ... + c_{n-2}c_n$$

S8.

$$\frac{c_r}{c_{r-1}} = \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = \frac{n-r+1}{r}$$

$$\Rightarrow \quad r \cdot \frac{c_r}{c_{r-1}} = n - r + 1$$

We put $r = 1, 2, ... n$ $\quad \frac{c_1}{c_0} = n, \quad 2\frac{c_2}{c_1} = n-1, \quad 3\frac{c_3}{c_2} = n-2$

$$n\frac{c_n}{c_{n-1}} = 1$$

On adding, we get

$$\frac{c_1}{c_0} + 2\frac{c_2}{c_1} + 3\frac{c_3}{c_2} + \ldots + \frac{nc_n}{c_{n-1}} = \frac{n(n+1)}{2}.$$

S9. We know that

$$(1 + x)^{2n} = (1 + x)^n (x + 1)^n$$

$$\Rightarrow \quad {}^{2n}c_0 + {}^{2n}c_1 x + \ldots + {}^{2n}c_r x^r + \ldots + {}^{2n}c_{2n} x^{2n}.$$

$$= [c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n] \cdot [c_0 x^n + c_1 x^{n-1} + \ldots + c_r x^{n-r} + c_{r+1} x^{n-r-1} + \ldots c_n]$$

Now comparing the coefficient of x^{n-r} we get

$${}^{2n}c_{n-r} = c_0 c_r + c_1 c_{r+1} + \ldots + c_{n-r} c_n$$

$$\Rightarrow \quad \frac{(2n)!}{(n-r)!(n+r)!} = c_0 c_r + c_1 c_{r+1} + \ldots + c_{n-r} c_n$$

S10. We know that

$$(1 + x)^{2n} = (1 + x)^n (x + 1)^n$$

$$\Rightarrow \quad {}^{2n}c_0 + {}^{2n}c_1 x + {}^{2n}c_2 x^2 + \ldots + {}^{2n}c_r x^r + \ldots + {}^{2n}c_{2n} x^{2n}$$

$$= [c_0 + c_1 x + c_2 x^2 + \ldots + c_n x^n] \cdot [c_0 x^n + c_1 x^{n-1} + c_2 x^{n-2} + \ldots + c_n]$$

Now comparing the coefficient of x^{n-1}, we get

$${}^{2n}c_{n-1} = c_0 c_1 + c_1 c_2 + \ldots + c_{n-1} c_n$$

$$\Rightarrow \quad \frac{(2n)!}{(n-1)!(n+1)!} = c_0 c_1 + \ldots + c_{n-1} c_n$$

$$\Rightarrow \quad \frac{2^n \cdot n! \cdot [1.3.5\ldots(2n-1)]}{(n-1)!(n+1)!} = c_0 c_1 + c_1 c_2 + \ldots + c_{n-1} c_n$$

S11.

$${}^2c_0 + 2^2\frac{c_1}{2} + 2^3\frac{c_2}{3} + \ldots + 2^{n+1}\frac{c_n}{n+1} = 2 + 2^2 \cdot \frac{n}{2} + 2^3 \cdot \frac{n(n-1)}{3.2} + \ldots + 2^{n+1}\frac{1}{n+1}$$

$$= \frac{1}{n+1}\left[(n+1)\cdot 2 + \frac{(n+1)\cdot n}{2!}2^2 + \frac{(n+1)(n)(n-1)}{3!}\cdot 2^3 + \ldots + 2^{n+1}\right]$$

$$= \frac{1}{n+1}[{}^{n+1}c_1 \cdot 2 + {}^{n+1}c_2 \cdot 2^2 + {}^{n+1}c_3 \cdot 2^3 + \ldots + {}^{n+1}c_{n+1} 2^{n+1}]$$

$$= \frac{1}{n+1}[(1+2)^{n+1} - 1] = \frac{3^{n+1} - 1}{n+1}.$$

S12. We know that

$$(1 + x)^{2n} = (1 + x)^n (x + 1)^n$$

$$\Rightarrow \quad {}^{2n}c_0 + {}^{2n}c_1 x + {}^{2n}c_2 x^2 + \ldots + {}^{2n}c_n x^n + \ldots + {}^{2n}c_{2n} x^{2n}$$

$$= [c_0 + c_1x + c_2x^2 + \dots + c_nx^n] \cdot [c_0x^n + c_1x^{n-1} + c_2x^{n-2} + \dots + c_n]$$

Now comparing the coefficient of x^n, we get

$$^{2n}c_n = c_0^2 + c_1^2 + \dots + c_n^2$$

$$\Rightarrow \quad \frac{(2n)!}{(n!)^2} = c_0^2 + c_1^2 + c_2^2 + \dots + c_n^2$$

S13. $c_0 + 3c_1 + 5c_2 + \dots + (2n + 1)c_n$

$$= c_0 + (c_1 + 2c_1) + (c_2 + 4c_2) + \dots + (c_n + 2nc_n)$$

$$= [c_0 + c_1 + c_2 + \dots + c_n] + [2c_1 + 4c_2 + \dots + 2nc_n]$$

$$= 2^n + 2[c_1 + 2c_2 + \dots + nc_n]$$

$$= 2^n + 2n\,2^{n-1} = (n + 1)2^n$$

S14. $c_0 + 2c_1 + 3c_2 + \dots + (n + 1)c_n$

$$= c_0 + (c_1 + c_1) + \dots + (c_n + nc_n)$$

$$= [c_0 + c_1 + c_2 + \dots + c_n] + [c_1 + 2c_2 + \dots + nc_n]$$

$$= 2^n + n2^{n-1}$$

$$= (n + 2)2^{n-1}$$

S15. $(c_0 + c_1)(c_1 + c_2)(c_2 + c_3)\dots(c_{n-1} + c_n) = \dfrac{c_0c_1c_2\dots c_{n-1}(n+1)^n}{n!}$

$$\Rightarrow \quad \left[\frac{c_0 + c_1}{c_0}\right]\left[\frac{c_1 + c_2}{c_1}\right]\left[\frac{c_2 + c_3}{c_2}\right]\dots\left[\frac{c_{n-1} + c_n}{c_{n-1}}\right] = \frac{(n+1)^n}{n!}$$

$$\Rightarrow \quad \left[1 + \frac{c_1}{c_0}\right]\left[1 + \frac{c_2}{c_1}\right]\left[1 + \frac{c_3}{c_2}\right]\dots\left[1 + \frac{c_n}{c_{n-1}}\right] = \frac{(n+1)^n}{n!}$$

We have

$$1 + \frac{c_r}{c_{r-1}} = 1 + \frac{^nc_r}{^nc_{r-1}}$$

$$= 1 + \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = 1 + \frac{n-r+1}{r} = \frac{n+1}{r}$$

we put $r = 1, 2, \dots, n$

$$1 + \frac{c_1}{c_0} = \frac{n+1}{1}, 1 + \frac{c_2}{c_1} = \frac{n+1}{2}, \dots$$

$$1 + \frac{c_n}{c_{n-1}} = \frac{n+1}{n}$$

On multiplying, we get

$$\left[1+\frac{c_1}{c_0}\right]\left[1+\frac{c_2}{c_1}\right]\ldots\ldots\left[1+\frac{c_n}{c_{n-1}}\right] = \frac{n+1}{1}\cdot\frac{n+1}{2}\ldots\frac{n+1}{n} = \frac{(n+1)^n}{n!}$$

$$\Rightarrow \quad (c_0 + c_1)(c_1 + c_2)(c_2 + c_3)\ldots(c_{n-1} + c_n) = \frac{c_0 c_1 \ldots c_{n-1}\cdot(n+1)^n}{n!}$$

Date: 12/10/2021

Q1. Identify the quantifier in the following statement and write the negation of statement "There exists a number which is equal to its square".

Q2. Show by contradiction method that $\sqrt{2}$ is an irrational number.

Q3. Write down the negation of the following compound statement's

(a) $\sqrt{3}$ is a rational number.

(b) All similar triangle are congruent.

Q4. Write the negation of the given statement.
$x + y = y + x$ and 13 is a prime number.

Q5. Write the truth value of the given statement.
"Every rectangle is a square and every square is a rectangle".

Q6. Write the negation of the following statement
"Asia is a continent"

Q7. Write the negation of statement
"$\sqrt{7}$ is rational"

Q8. What is truth value of statement
"100 is divisible by 3, 11, and 5".

Q9. By giving counter example show that the following statement is false. "If n is an odd integer then it is prime .

Q10. Write the negation of the following statement: Both the diagonals of a rectangle have the same length.

Q11. Write the negation of the following statement: $\sqrt{7}$ is rational.

Q12. Write the negation of the following statement: Australia is a continent.

Q13. Write the negation of the following statement: There does not exist a quadrilateral which has all its sides equal.

Q14. Write the negation of the following statement: Every natural number is greater than 0.

Q15. Write the negation of the following statement: The sum of 3 and 4 is 9.

Q16. From the following statements, determine whether an inclusive "Or" or exclusive "Or" is used. Give reasons for your answer.

To enter a country, you need a passport or a voter registration card.

Q17. From the following statements, determine whether an inclusive "Or" or exclusive "Or" is used. Give reasons for your answer.

The school is closed if it is a holiday or Sunday.

Q18. From the following statements, determine whether an inclusive "Or" or exclusive "Or" is used. Give reasons for your answer.

Two lines intersect at a point or are parallel..

Q19. From the following statements, determine whether an inclusive "Or" or exclusive "Or" is used. Give reasons for your answer.

Students can take French or Sanskrit as their third language.

Q20. Identify the type of "Or" used in the following statements and check whether the statements are true or false:

To enter into a public library children need an identity card from the school or a letter from the school authorities.

Q21. Identify the type of "Or" used in the following statements and check whether the statements are true or false:

A rectangle is a quadrilateral or a 5-sided polygon.

Q22. Given below are two pairs of statements. Combine these two statements using "if and only if".

p : If a rectangle is a square, then all its four sides are equal.

q : If all the four sides of a rectangle are equal, then the rectangle is a square.

Q23. Given below are two pairs of statements. Combine these two statements using "if and only if".

p : If the sum of digits of a number is divisible by 3, then the number is divisible by 3.

q : If a number is divisible by 3, then the sum of its digits is divisible by 3.

Q24. Write the negation of the following statements:

$\sqrt{2}$ *is not a complex number.*

Q25. Write the negation of the following statements:

Chennai is the capital of Tamil Nadu.

Q26. For the given statements identify the necessary and sufficient conditions.

t : If you drive over 80 km per hour, then you will get a fine.

Q27. Write the negation of the following statements:

s : All students study mathematics at the elementary level.

Q28. Write the negation of the following statements:

r : All birds have wings.

Q29. Write the negation of the following statements:

q : There exists a rational number x such that $x^2 > 2$.

Q30. Write the negation of the following statements:

p : For every real number x, $x^2 > x$.

Q31. Check whether "Or" used in the following compound statement is exclusive or inclusive? Write the component statements of the compound statements and use them to check whether the compound statement is true or not. Justify your answer.

t : You are wet when it rains or you are in a river.

Q32. Verify by the method of contradiction.

p : $\sqrt{7}$ *is irrational.*

Q33. Are the following pairs of statements negation of each other:

(i) The number x is not a rational number.

The number x is not an irrational number.

(ii) The number x is a rational number.

The number x is an irrational number.

Q34. Write the negation of the following statements:

Every natural number is an integer.

Q35. Write the negation of the following statements:

The number 2 is greater than 7.

Q36. Write the negation of the following statements:

All triangles are not equilateral triangles.

Q37. Identify the quantifier in the following statements and write the negation of the statements:

For every real number x, x is less than x + 1.

Q38. Identify the quantifier in the following statements and write the negation of the statements:

There exists a number which is equal to its square.

Q39. Identify the quantifier in the following statements and write the negation of the statements:

There exists a capital for every state in India.

Q40. Write the negation of the component of the following statements:

$x = 2$ *and* $x = 3$ *are the roots of the equation* $3x^2 - x - 10 = 0$.

Q41. Write the negation of the component of the following statements:

The sand heats up quickly in the Sun and does not cool down fast at night.

Q42. Write the negation of the component of the following statements:

Square of an integer is positive or negative.

Q43. Write the negation of the component of the following statements:

All rational numbers are real and all real numbers are not complex.

Q44. Check whether the following pair of statements are negation of each other. Give reasons for your answer.

(i) $x + y = y + x$ is true for every real numbers x and y.

(ii) There exists real numbers x and y for which $x + y = y + x$.

Q45. Write the negation of the following statements:

q : All cats scratch.

Q46. Write the negation of the following statements:

s : There exists a number x such that $0 < x < 1$.

Q47. Write the negation of the following statements:

r : For every real number x, either $x > 1$ or $x < 1$.

Q48. Show that the statement "For any real numbers a and b, $a^2 = b^2$ implies that $a = b$" is not true by giving a counter-example.

Q49. Write the negation of the following statements:

p : For every positive real number x, the number $x - 1$ is also positive.

Q50. Using the words "necessary and sufficient" rewrite the statement "The integer n is odd if and only if n^2 is odd". Also, check whether the statement is true.

Q51. Write the contrapositive and converse of the following statements:

(i) If x is a prime number, then x is odd.

(ii) If the two lines are parallel, then they do not intersect in the same plane.

(iii) Something is cold implies that it has low temperature.

(iv) You cannot comprehend geometry if you do not know how to reason deductively.

(v) x is an even number implies that x is divisible by 4.

Q52. State the converse and contrapositive of each of the following Statement:

(i) p : A positive integer is prime only if it has no divisors other than 1 and itself.

(ii) q : I go to a beach whenever it is a sunny day.

(iii) r : If it is hot outside, then you feel thirsty.

Q53. Show that the statement:

p : "If x is a real number such that $x^3 + 4x = 0$, the x is 0" is true by:

(i) direct method, (ii) method of contradiction, (iii) method of contrapositive.

Q54. Given statements in (a) and (b). Identify the statements given below as contrapositive or converse of each other.

(a) (i) If you live in Delhi, then you have winter clothes.

(ii) If you do not have winter clothes, then you do not live in Delhi.

(b) (i) If a quadrilateral is a parallelogram, then its diagonals bisect each other.

(ii) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

Date: 12/10/2021

S1. The quantifier in the given statement is "there exists" and the negation is "There does not exists a number which is equal to its square".

S2. Let if possible $\sqrt{2}$ is a rational number.

$\Rightarrow \quad \sqrt{2} = \dfrac{a}{b}$, $\quad a$ and b are coprimes, $b \neq 0$.

$\Rightarrow \quad a = \sqrt{2}\, b$.

$\Rightarrow \quad a^2 = 2\, k,\ k \in z$.

$\Rightarrow \quad a^2 = 4k^2$

$\Rightarrow \quad 2\, b^2 = 4\, k^2 \quad \Rightarrow \quad b^2 = 2\, k^2$

$\Rightarrow \quad b^2$ is even $\quad \Rightarrow \quad b$ is even

We have seen that a and b both are even. It means a and b, both have common factor 2 it contradicts our assumption that a and b have no common factor.

$\Rightarrow$ Our assumption that $\sqrt{2}$ is rational is incorrect

$\Rightarrow$ $\sqrt{2}$ is irrational.

S3. (a) "It is not true that, $\sqrt{3}$ is a rational number".

(b) "It is not true that similar triangles are congruent".

S4. Let $p : x + y = y + x$

and q : 13 is a prime number.

Then the conjunction is given by $p \wedge q$.

Now $\sim p : x + y \neq y + x$

$\sim q$: 13 is not a prime number.

Therefore, negation of $p \wedge q$ is given by $\sim (p \wedge q) : x + y \neq y + x$ or 13 is not a prime number.

S5. Truth value is F as first component statement namely every rectangle is a square is false.

S6. "It is not true that Asia is a continent".

S7. "It is not true as $\sqrt{7}$ is rational".

S8. Let p :100 is divisible by 3 "F"

q: 100 is divisible by 11 "F"

r: 100 is divisible by 5 "T"
Truth value is F as first two component statement are False.

S9. "If we taken $n = 9 \Rightarrow n$ is odd".

$n = 9 = 3 \times 3$ is not a prime number.

$\Rightarrow$ $n = 9$ is the counter example to show that the given statement is false.

S10. This statement says that in a rectangle, both the diagonals have the same length. This means that if you take any rectangle, then both the diagonals have the same length. The negation of this statement is

There is atleast one rectangle whose both diagonals do not have the same length.

S11. The negation of the statement may also be written as

It is not the case that $\sqrt{7}$ is rational.

This can also be rewritten as

$\sqrt{7}$ is not rational.

S12. The negation of the statement is

It is false that Australia is a continent.

This can also be rewritten as

Australia is not a continent.

We know that this statement is false.

S13. The negation of the statement is

It i not the case that there does not exist a quadrilateral which has all its sides equal.

This also means the following:

There exists a quadrilateral which has all its sides equal.

This statement is true because we know that square is a quadrilateral such that its four sides are equal.

S14. The negation of the statement is

It is false that every natural number is greater than 0.

This can be rewritten as

There exists a natural number which is not greater than 0.

This is a false statement.

S15. The negation is

It is false that the sum of 3 and 4 is 9.

This can be written as

The sum of 3 and 4 is not equal to 9.

This statement is true.

S16. Here "Or" is inclusive since a person can have both a passport and a voter registration card to enter a country.

S17. Here also "Or" is inclusive since school is closed on holiday as well as on Sunday.

S18. Here "Or" is exclusive because it is not possible for two lines to intersect and parallel together.

S19. Here also "Or" is exclusive because a student cannot take both French and Sanskrit.

S20. The component statements are

p : *To get into a public library children need an identity card*.

q : *To get into a public library children need a letter from the school authorities.*

Children can enter the library if they have either of the two, an identity card or the letter, as well as when they have both. Therefore, it is inclusive "Or" the compound statement is also true when children have both the card and the letter.

S21. Here "Or" is exclusive. When we look at the component statements, we get that the statement is true.

S22. A rectangle is a square if and only if all its four sides are equal.

S23. A number is divisible by 3 if and only if the sum of its digits is divisible by 3.

S24. $\sqrt{2}$ is a complex number.

S25. Chennai is not the capital of Tamil Nadu.

S26. Let p and q denote the statements:

p : You drive over 80 km per hour.

q : You will get a fine.

The implication if p, then q indicates that p is sufficient for q. That is driving over 80 km per hour is sufficient to get a fine.

Here the sufficient condition is "driving over 80 km per hour".

Similalry, if p, then q also indicates that q is necessary for p. That is

When you drive over 80 km per hour, you will necessarily get a fine.

Here, the necessary condition is "getting a fine".

S27. The negation of the given statement is

$\sim s$: There exists a student who does not study mathematics at the elementary level.

S28. The negation of the statement is

$\sim r$: There exists a bird which have no wings.

S29. Negation of q is "It is false that q". Thus $\sim q$ is the statement.

$\sim q$: There does not exists a rational number x such that $x^2 > 2$.

This statement can be rewritten as

$\sim q$: For all real numbers x, $x^2 \neq 2$.

S30. The negation of p is "It is false that p is" which means that the condition $x^2 > x$ does not hold for all real numbers. This can be expressed as

$\sim p$: There exists a real number x such that $x^2 < x$.

S31. "Or" used in the given statement is inclusive because it is possible that it rains and you are in the river.

The compoment statements of the given statement are

p : *You are wet when it rains.*

q : *You are wet when you are in a river.*

Here both the component statements are true and therefore, the compound statement is true.

S32. In this method, we assume that the given statement is false. That is we assume that $\sqrt{7}$ is rational. This means that there exists positive integers a and b such that $\sqrt{7} = \frac{a}{b}$, where a and b have no common factors. Squaring the equation, we get $7 = \frac{a^2}{b^2} \Rightarrow a^2 = 7b^2 \Rightarrow 7$ divides a. Therefore, there exists an integer c such that $a = 7c$. Then $a^2 = 49c^2$ and $a^2 = 7b^2$.

Hence,$7b^2 = 49c^2 \Rightarrow b^2 = 7c^2 \Rightarrow 7$ divides b. But we have already shown that 7 divides a. This implies that 7 is a common factor of both of a and b which contradicts our earlier assumption that a and b have no common factors. This shows that the assumption $\sqrt{7}$ is rational is wrong. Hence, the statement $\sqrt{7}$ is irrational is true.

S33. (i) The negation of the first statement is "the number x is a rational number". Which is the same as the second statement. This because if that the number is not irrational, it is a rational.

Therefore, the given pairs are negations of each other.

(ii) The negation of the first statement is "x is an irrational number" which is the same as the second statement. Therefore, the pairs are negations of each other.

S34. Every natural number is not an integer.

S35. The number 2 is not greater than 7.

S36. All triangles are equilateral triangles.

S37. Quantifier is "For every".

The negation of the statement is: There exists a real number x such that x is not less than $x + 1$.

S38. Quantifier is "There exists".

Negation is: There does not exist a number which is equal to its square.

S39. Quantifier is "There exists".

The negation of the statement is: There exists a state in India which does not have a capital.

S40. Negation of the components are:

$x = 2$ is not a root of the equation $3x^2 - x - 10 = 0$.

$x = 3$ is not a root of the equation $3x^2 - x - 10 = 0$.

S41. Negation of the components are:

The sand does not heat up quickly in the Sun.

The sand cools down fast at night.

S42. Negation of the components are:

Square of an integer is not positive.

Square of an integer is not negative.

S43. Negation of the components are:

All rational numbers are not real.

All real numbers are complex.

S44. The following pair of statements are not negation of each other.

The negation of the statement in (i) is "There exists real number x and y for which $x + y \neq y + x$", instead of the statement given in (ii).

S45. q : All cats scratch.

$\sim q$: There exists a cat which does not scratch.

S46. s : There exists a number x such that $0 < x < 1$.

$\sim$ s : There does not exist a real number x such that $0 < x < 1$.

S47. r : For every real x, either $x > 1$ or $x < 1$.

$\sim r$: There exists a number x such that neither $x > 1$ nor $x < 1$, *i.e.*, $x = 1$.

S48. Let $a = 1, \quad b = -1$

and $a^2 = b^2$ but $a \neq b$

Thus, we observe that the given statement is not true.

S49. p : For every positive real number x, the number $x - 1$ is also positive.

$\sim p$: There exists a positive real number x such that x – 1 is not positive.

S50. The necessary and sufficient condition that the integer n be odd is n^2 must be odd. Let p and q denote the statements

p : The integer n is odd.

q : n^2 is odd.

To check the validity of "p if and only if q", we have to check whether "if p then q" and "if q then p" is true.

Case 1 : If p, then q

If p, then q is the statement:

If the integer n is odd, then n^2 is odd. We have to check whether this statement is true. Let us assume that n is odd. Then $n = 2k + 1$ when k is an integer. Thus

$$n^2 = (2k + 1)^2$$
$$= 4k^2 + 4k + 1$$

Therefore, n^2 is one more than an even number and hence is odd.

Case 2 : If q, then p

If q, then p is the statement:

If n is an integer and n^2 is odd, then n is odd.

We have to check whether this statement is true. We check this by contrapositive method. The contrapositive of the given statement is:

If n an even integer, then n^2 is an even integer.

n is even implies that $n = 2k$ for some k. Then $n^2 = 4k^2$. Therefore, n^2 is even.

S51. (i) **Given statement:** If x is a prime number, then x is odd.

The contrapositive statement: If a number x is not odd, then x is not a prime number.

The converse statement: If a number x is odd, then it is a prime number.

(ii) **Given statement:** If the two lines are parallel, then they do not intersect in the same plane.

The contrapositive statement: If two lines intersect in the same plane, then they are not parallel.

The converse statement: If two lines do not intersect in the same plane, then they are parallel.

(iii) **Given statement:** Something is cold implies that it has low temperature.

The contrapositive statement: If some things is not at low temperature, then it is not cold.

The converse statement: If something is at low temperature, then it is cold.

(iv) **Given statement:** You cannot comprehend geometry if you do not know how to reason deductively.

The contrapositive statement: If you know how to reason deductively, then you cannot comprehend geometry.

The converse statement: If you do not know how to reason deductively, then you cannot comprehend geometry.

(v) **Given statement:** x is an even number implies that x is divisible by 4.

It can be written as: "If x is an even number, then x is divisible by 4".

The contrapositive statement: If x is not divisible by 4, then x is not an even number.

The converse statement: If x is divisible by 4, then x is an even number.

S52. (i) p : A positive integer is prime only if it has no divisors other than 1 and itself.

The statement p can be written as "If a positive integer is prime, then it has no divisors other than 1 and itself".

Its converse statement is: "If a positive number has no divisors other than 1 and itself then it is a prime."

The contrapositive of the statement is: "If a positive integer has divisors other than 1 and itself, then it is not prime."

(ii) q : I go to a beach whenever it is a sunny day.

The given statement can be written as: It is a sunny day, then I go to beach.

Converse statement is: "If I go to a beach, then it is a sunny day.

Contrapositive statement: If I do not go to a beach, then it is not a sunny day.

(iii) r: If it is hot outside, then you feel thristy.

The converse statement is: If you feel thirsty, then it is hot out side.

The contrapositive statement: If you do not feel thirsty, then it is not hot out side.

S53. (i) **Direct method:**

$$x^3 + 4x = 0 \Rightarrow x(x^2 + 4) = 0$$

Now, $x^2 + 4 \neq 0$ as $x \in R$

Hence, $x = 0$

(ii) **Method of contradiction:**

Let $x \neq 0$

and let $x = p$, $p \in R$ is a root of

$$x^3 + 4x = 0$$

Therefore, $p^3 + 4p = 0$

or $p(p^2 + 4) = 0$

as $p = 0$

Thus $p^2 + 4 = 0$ which is not possible.

Therefore, our supposition is wrong.

Hence, $p = 0$ or $x = 0$.

(iii) **Method of contrapositive:** p is not true.

$\Rightarrow$ Let $x = 0$ is not true

$\Rightarrow$ Let $x = p \times 0$

Therefore, $p^3 + 4p = 0$

p being the root of $x^2 + 4 = 0$

or $p(p^2 + 4) = 0$

Now, $p = 0$

Also, $p^2 + 4 = 0$

$\Rightarrow$ $p(p^2 + 4) \neq 0$

if p is not true

Hence, $x = 0$ is the root of $x^3 + 4x = 0$.

S54. (a) (i) Contrapositive statement. (ii) Converse statement.

(b) (i) Contrapositive statement. (ii) Converse statement.